Pass-Through and Mergers

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These notes describe how equilibrium pass-through rates for consumer-facing firms change following mergers with cost synergies.

1 Setup

The economy exists in partial equilibrium and consists of a set I of firms and a representative consumer. Each firm $i \in I$ produces a differentiated and imperfectly substitutable good for final consumption. Let $p := (p_i)_{i \in I}$ denote the vector of prices, and let $c := (c_i)_{i \in I}$ denote the vector of constant marginal costs. The representative consumer's demand function for good $i \in I$ is denoted $y_i(p)$. I assume that y_i is continuous on $\mathbb{R}_+^I$, strictly decreasing in p_i, and twice continuously differentiable on $\{p \mid y_i(p) > 0\}$. Let $y(p) := (y_i(p))_{i \in I}$ denote the vector of outputs for the downstream firms. Finally, define the profit functions by

$$\pi_i(p) := (p_i - c_i)\, y_i(p) \quad i \in I. \tag{1.1}$$

The solution concept is Bertrand-Nash, so that a *pre-merger equilibrium* is a price vector p such that each firm chooses its price to maximize its profits, holding the prices of the remaining firms constant:

$$p_i \in \arg\max_{\tilde{p}_i \geq 0} \pi_i(\tilde{p}_i, p_{-i}) \quad i \in I. \tag{1.2}$$

I consider a merger between two firms $m, m' \in I$, and I abuse notation by using $M := \{m, m'\}$ as the "index" of the merged firm. Following the merger, the firm M realizes new marginal costs

$$\hat{c}_M := (\hat{c}_m, \hat{c}_{m'}) \leq (c_m, c_{m'}) =: c_M. \tag{1.3}$$

Let $\hat{c} := (\hat{c}_i)_{i \in I}$ denote the post-merger vector of constant marginal costs, where $\hat{c}_i = c_i$ for

$i \notin M$. Define firm M's post-merger profit function by

$$\hat{\pi}_M(\hat{p}) := \sum_{m \in M} (\hat{p}_m - \hat{c}_m)\, y_m(\hat{p}). \tag{1.4}$$

The profit function for an "outsider firm" $i \in I \setminus M$ is unchanged after the merger. A *post-merger equilibrium* is a price vector $\hat{p}$ such that each firm chooses its price (or prices, in the case of the merged firm) to maximize its profits, holding other prices constant:

$$\hat{p}_i \in \arg\max_{\tilde{p}_i \geq 0} \pi_i(\tilde{p}_i, \hat{p}_{-i}) \qquad i \in I, \tag{1.5}$$

$$(\hat{p}_m, \hat{p}_{m'}) \in \arg\max_{\tilde{p}_m, \tilde{p}_{m'} \geq 0} \pi_M(\tilde{p}_m, \tilde{p}_{m'}, \hat{p}_{-M}). \tag{1.6}$$

In what follows, I assume the pre- and post-merger economies permit interior equilibria in which the price and output of each good is strictly positive. So as to ensure well-behaved comparative statics, I also assume that the second order conditions are satisfied strictly for each firm in equilibrium.

2 Definitions and General Expressions

Pre-Merger Equilibrium. Pre-merger equilibrium prices p must satisfy the interior first order conditions

$$0 = y_i + (p_i - c_i)\frac{\partial y_i}{\partial p_i} \qquad i \in I. \tag{2.1}$$

Differentiating this condition, we can derive the partial equilibrium (p_{-i}-constant) pass-through rates

$$\rho_{ij} := \frac{\partial p_i}{\partial c_j} \tag{2.2}$$

$$= \mathbb{1}[i = j]\frac{\frac{\partial y_i}{\partial p_i}}{2\frac{\partial y_i}{\partial p_i} + (p_i - c_i)\frac{\partial^2 y_i}{\partial p_i^2}} \qquad j \in I. \tag{2.3}$$

We can also calculate the matrix of best-response derivatives:

$$B_{ij} := \mathbb{1}[i \neq j] \frac{\partial p_i}{\partial p_j} \tag{2.4}$$

$$= -\mathbb{1}[i \neq j] \frac{\frac{\partial y_i}{\partial p_j} + (p_i - c_i) \frac{\partial^2 y_i}{\partial p_j \partial p_i}}{2\frac{\partial y_i}{\partial p_i} + (p_i - c_i) \frac{\partial^2 y_i}{\partial p_i^2}} \qquad j \in I. \tag{2.5}$$

The matrix of equilibrium pass-through rates P then satisfies

$$\mathrm{P} := \frac{dp}{dc} = (\mathbf{I} - B)^{-1} \rho. \tag{2.6}$$

Post-Merger Equilibrium. After the merger between firms m and m', equilibrium prices $\hat{p}$ must satisfy the interior first order conditions

$$0 = y_i + (\hat{p}_i - c_i) \frac{\partial y_i}{\partial p_i} \quad i \in I \setminus M, \tag{2.7}$$

$$0 = y_m + (\hat{p}_m - \hat{c}_m) \frac{\partial y_m}{\partial p_m} + (\hat{p}_{m'} - \hat{c}_{m'}) \frac{\partial y_{m'}}{\partial p_m} \quad m \in M. \tag{2.8}$$

The partial equilibrium pass-through rates and best-response derivatives are the same as in the pre-merger case for outsider firms $i \in I \setminus M$. For $m \in M$, the "partial equilibrium" ($\hat{p}_{-M}$- and $\hat{p}_{m'}$-constant) pass-through rates are

$$\hat{\rho}_{mj} := \frac{\partial \hat{p}_m}{\partial \hat{c}_j} \tag{2.9}$$

$$= \mathbb{1}[j = m] \frac{\frac{\partial y_m}{\partial p_m}}{2\frac{\partial y_m}{\partial p_m} + (\hat{p}_m - \hat{c}_m) \frac{\partial^2 y_m}{\partial p_m^2} + (\hat{p}_{m'} - \hat{c}_{m'}) \frac{\partial^2 y_{m'}}{\partial p_m^2}} \tag{2.10}$$

$$+ \mathbb{1}[j = m'] \frac{\frac{\partial y_{m'}}{\partial p_m}}{2\frac{\partial y_m}{\partial p_m} + (\hat{p}_m - \hat{c}_m) \frac{\partial^2 y_m}{\partial p_m^2} + (\hat{p}_{m'} - \hat{c}_{m'}) \frac{\partial^2 y_{m'}}{\partial p_m^2}}. \tag{2.11}$$

We also have the best-response derivatives

$$\hat{B}_{mj} := \mathbb{1}[i \neq j] \frac{\partial \hat{p}_i}{\partial \hat{p}_j} \tag{2.12}$$

$$= -\mathbb{1}[i \neq j] \frac{\frac{\partial y_m}{\partial p_j} + (\hat{p}_m - \hat{c}_m) \frac{\partial^2 y_m}{\partial p_j \partial p_m} + (\hat{p}_{m'} - \hat{c}_{m'}) \frac{\partial^2 y_{m'}}{\partial p_i \partial p_m} + \mathbb{1}[j = m'] \frac{\partial y_{m'}}{\partial p_m}}{2\frac{\partial y_m}{\partial p_m} + (\hat{p}_m - \hat{c}_m) \frac{\partial^2 y_m}{\partial p_m^2} + (\hat{p}_{m'} - \hat{c}_{m'}) \frac{\partial^2 y_{m'}}{\partial p_m^2}}. \tag{2.13}$$

The post-merger matrix of equilibrium pass-through rates $\hat{\mathrm{P}}$ then satisfies

$$\hat{\mathrm{P}} := \frac{d\hat{p}}{d\hat{c}} = \left(\mathbf{I} - \hat{B}\right)^{-1} \hat{\rho}. \tag{2.14}$$

3 Special Case: Linear Demand, Merge to Monopoly

I first consider a special case in which the demand system y is linear and there are only two firms pre-merger, $|I| = 2$.

3.1 Price Effects

Linear demand implies that the pre-merger pass-through rates and best response derivatives simplify to

$$\rho_{ij} = \frac{\mathbb{1}[i = j]}{2}, \tag{3.1}$$

$$B_{ij} = -\mathbb{1}[i \neq j] \frac{\frac{\partial y_i}{\partial p_j}}{2\frac{\partial y_i}{\partial p_i}} = \mathbb{1}[i \neq j] \frac{D_{ij}}{2}, \tag{3.2}$$

$$\mathrm{P}_{ij} = \frac{1/2}{1 - \frac{D_{12}}{2}\frac{D_{21}}{2}} \left(\mathbb{1}[i = j] + \mathbb{1}[i \neq j] \frac{D_{ij}}{2} \right). \tag{3.3}$$

Here I have defined the *diversion ratio* from i to j:

$$D_{ij} := -\mathbb{1}[i \neq j] \frac{\partial y_i / \partial p_j}{\partial y_i / \partial p_i}. \tag{3.4}$$

I assume that all goods are imperfectly substitutable, so this quantity is strictly positive for all $i \neq j$. Notably, The best response derivative B_{ij} is strictly increasing in D_{ij}, while the equilibrium pass-through rates P_{ii} and P_{ij} for $i \neq j$ are all strictly increasing in D_{ij} and D_{ji}. Hence prices are most sensitive to changes in marginal costs in industries with highly substitutable goods.

After the merger, we have

$$\hat{\rho}_{mj} = \frac{1}{2} \left(1 - \mathbb{1}\left[j = m'\right] (D_{mm'} + 1)\right), \tag{3.5}$$

$$\hat{B}_{mj} = -\mathbb{1}\left[j = m'\right] \frac{\frac{\partial y_m}{\partial p_{m'}} + \frac{\partial y_{m'}}{\partial p_m}}{2\frac{\partial y_m}{\partial p_m}} = \mathbb{1}\left[j = m'\right] D_{mj}, \tag{3.6}$$

$$\hat{\mathrm{P}}_{mj} = \mathbb{1}[j = m] \frac{1}{2}. \tag{3.7}$$

These expressions imply several important "comparative dynamics" between the pre- and post-merger economies. First, the partial equilibrium pass-through rate of the price of good m with respect to its own marginal cost remains unchanged:

$$\hat{\rho}_{mm} = \hat{\rho}_{mm'} = \frac{1}{2}. \tag{3.8}$$

However, the partial equilibrium pass-through rate of the price of good m with respect to the marginal cost of the other good m' declines:

$$\hat{\rho}_{mm'} < \rho_{mm'} = 0. \tag{3.9}$$

Intuitively, this pass-through rate becomes negative because an increase in the marginal cost of m' implies that it is profitable to lower p_m and divert demand from m' to m ("Edgeworth-Salinger effect"). The decline in this pass-through rate is larger for larger diversion ratios $D_{mm'}$.

Second, the best response derivatives increase following the merger:

$$\hat{B}_{mm'} > B_{mm'}. \tag{3.10}$$

This holds because an increase in $p_{m'}$ affects the optimality condition for p_m in two ways. As before the merger, an increase in $p_{m'}$ raises the level of demand y_m, inducing an increase in p_m to extract rents from inframarginal consumers. After the merger, an increase in $p_{m'}$ also makes it profitable to raise p_m and divert demand from m to m', since sales of m' now earn a higher margin (which is internalized by the merged firm). The increase in the best response derivative is larger for larger diversion ratios $D_{mm'}$.

Finally, we observe that the equilibrium pass-through rate of the price of good m with respect to its own marginal cost declines:

$$\hat{\mathrm{P}}_{mm} < \mathrm{P}_{mm}. \tag{3.11}$$

Although higher best response derivatives tend toward amplifying the equilibrium pass-through rate, this effect is dominated by the merged firm's incentive to decrease $p_{m'}$ in response to an increase in c_m. Hence the "Le Chatelier ratio" (ratio of the partial equilibrium pass-through to the equilibrium pass-through) increases after the merger:

$$1 = \frac{\hat{\rho}_{mm}}{\hat{\mathrm{P}}_{mm}} > \frac{\rho_{mm}}{\mathrm{P}_{mm}} = 1 - \frac{D_{12}}{2}\frac{D_{21}}{2}. \tag{3.12}$$

Moreover, the decline in the mm equilibrium pass-through rate is larger for larger diversion ratios D_{12} and D_{21}. For a similar reason, the equilibrium pass-through rate of the price of good m with respect to the marginal cost of m' also declines:

$$0 = \hat{\mathrm{P}}_{mm'} < \mathrm{P}_{mm'} = \frac{\frac{D_{mm'}}{4}}{1 - \frac{D_{12}}{2}\frac{D_{21}}{2}}. \tag{3.13}$$

Again, this decline is larger for larger diversion ratios D_{12} and D_{21}. Hence all pass-through rates strictly decline, so $0 \leq \hat{\mathrm{P}} \ll \mathrm{P}$.

3.2 Output Effects

We can also determine how equilibrium outputs respond to changes in marginal costs. Before the merger, we can calculate

$$\frac{dy_i}{dc_j} = \left[\frac{\partial y}{\partial p}\mathrm{P}\right]_{ij} \tag{3.14}$$

$$= \frac{1/2}{1 - \frac{D_{12}}{2}\frac{D_{21}}{2}}\left\{\mathbb{1}\left[i = j\right]\left(\frac{\partial y_i}{\partial p_i} + \frac{\partial y_i}{\partial p_k}\frac{D_{ki}}{2}\right) + \mathbb{1}\left[i \neq j\right]\left(\frac{\partial y_i}{\partial p_i}\frac{D_{ij}}{2} + \frac{\partial y_i}{\partial p_j}\right)\right\}. \tag{3.15}$$

Holding $\partial y_i / \partial p_i$ constant, the output of i is more responsive to the marginal costs of both goods for higher diversion ratios D_{12} and D_{21}.

After the merger, we have

$$\frac{d\hat{y}_m}{d\hat{c}_j} = \left[\frac{\partial y}{\partial p}\hat{\mathrm{P}}\right]_{mj} \tag{3.16}$$

$$= \frac{1}{2}\left\{\mathbb{1}\left[j = m\right]\frac{\partial y_m}{\partial p_m} + \mathbb{1}\left[j = m'\right]\frac{\partial y_m}{\partial p_{m'}}\right\}. \tag{3.17}$$

We observe that equilibrium output of m is more responsive to a change in the marginal cost of m after the merger:

$$\frac{dy_m}{dc_m} = \frac{1 - D_{mm'}\frac{D_{m'm}}{2}}{1 - \frac{D_{12}}{2}\frac{D_{21}}{2}}\frac{1}{2}\frac{\partial y_m}{\partial p_m} > \frac{1}{2}\frac{\partial y_m}{\partial p_m} = \frac{d\hat{y}_m}{d\hat{c}_m} \tag{3.18}$$

Moreover, holding $\partial y_m / \partial p_m$ constant, the decline in the equilibrium output derivative is larger for larger diversion ratios D_{12} and D_{21}:

$$\frac{d\hat{y}_m}{d\hat{c}_m} - \frac{dy_m}{dc_m} = \frac{1}{2}\frac{\partial y_m}{\partial p_m}\frac{\frac{D_{12}}{2}\frac{D_{21}}{2}}{1-\frac{D_{12}}{2}\frac{D_{21}}{2}}. \tag{3.19}$$

For intuition, note that we can write

$$\frac{d\hat{y}_m}{d\hat{c}_m} - \frac{dy_m}{dc_m} = \frac{\partial y_m}{\partial p_m}\left(\hat{\mathrm{P}}_{mm} - \mathrm{P}_{mm}\right) + \frac{\partial y_m}{\partial p_{m'}}\left(\hat{\mathrm{P}}_{m'm} - \mathrm{P}_{m'm}\right). \tag{3.20}$$

As we have seen, both parenthetical terms are negative because all equilibrium pass-through rates fall after the merger. The decline in the mm equilibrium pass-through rate tends to make demand for m less sensitive to the marginal cost of m, while the decline in the $m'm$ pass-through rate works in the opposite direction. With linear demand and a merger to monopoly, the decline in the mm equilibrium pass-through rate is smaller than the $D_{mm'}$-weighted decline in the $m'm$ pass-through rate, so the latter effect dominates:

$$\hat{\mathrm{P}}_{mm} - \mathrm{P}_{mm} > D_{mm'}\left(\hat{\mathrm{P}}_{m'm} - \mathrm{P}_{m'm}\right). \tag{3.21}$$

Intuitively, the merged firm finds it profitable to divert a larger amount of demand from m to m' in response to an increase in the marginal cost of m than would have happened in equilibrium before the merger. By the same token, the equilibrium output of m is also more responsive to a change in the marginal cost of m' after the merger:

$$\frac{dy_m}{dc_{m'}} = \frac{1/2}{1-\frac{D_{12}}{2}\frac{D_{21}}{2}}\frac{1}{2}\frac{\partial y_m}{\partial p_{m'}} < \frac{1}{2}\frac{\partial y_m}{\partial p_{m'}} = \frac{d\hat{y}_m}{d\hat{c}_{m'}}. \tag{3.22}$$

The inequality holds because diversion ratios are bounded above by one. Hence all outputs become more responsive to changes in marginal costs in equilibrium, so $0 \leq \mathrm{abs}(dy/dc) \ll \mathrm{abs}(d\hat{y}/d\hat{c})$.

3.3 Synergies

Suppose that the merger yields a proportional reduction $E_m \in [0,1]$ in the marginal cost of each good m, so that

$$\hat{c}_m = (1-E_m)\,c_m \qquad m \in M. \tag{3.23}$$

In some cases, for example when c_m represents the price of a bundle of input goods, the post-merger equilibrium pass-through rates of c (not $\hat{c}$) to $\hat{p}$ are important. These are given by

$$\frac{d\hat{p}}{dc} = \hat{\mathrm{P}}\,\mathrm{diag}\,(1-E)\,. \tag{3.24}$$

Merger-induced cost synergies naturally dampen the equilibrium response of prices to a change in the input costs c. Hence

$$0 \leq \frac{d\hat{p}}{dc} \leq \hat{\mathrm{P}} \ll \mathrm{P}. \tag{3.25}$$

Similarly, synergies dampen the equilibrium response of outputs to a change in the input costs:

$$\frac{d\hat{y}}{dc} = \frac{d\hat{y}}{d\hat{c}}\,\mathrm{diag}\,(1-E) = \frac{\partial y}{\partial p}\hat{\mathrm{P}}\,\mathrm{diag}\,(1-E) \Rightarrow \mathrm{abs}\left(\frac{d\hat{y}}{dc}\right) \leq \mathrm{abs}\left(\frac{d\hat{y}}{d\hat{c}}\right). \tag{3.26}$$

However, since outputs are more responsive to marginal costs $\hat{c}$ after the merger, it is generally ambiguous whether outputs respond more or less to input costs c after the merger. For example, y_m is more responsive to c_m after the merger if and only if

$$\frac{d\hat{y}_m}{dc_m} \leq \frac{dy_m}{dc_m} \iff E_m \leq \frac{\frac{D_{12}}{2}\frac{D_{21}}{2}}{1-\frac{D_{12}}{2}\frac{D_{21}}{2}} \tag{3.27}$$

Denote the upper bound on the right side of the inequality by $\bar{E}$. Notably, this ceiling is symmetric across m and m', and it is increasing in the diversion ratios D_{12} and D_{21}. Now y_m is more responsive to $c_{m'}$ after the merger if and only if

$$\frac{d\hat{y}_m}{dc_{m'}} \geq \frac{dy_m}{dc_{m'}} \iff E_{m'} \leq \frac{\frac{1}{2}-\frac{D_{12}}{2}\frac{D_{21}}{2}}{1-\frac{D_{12}}{2}\frac{D_{21}}{2}}. \tag{3.28}$$

Denote the upper bound on the right side of the inequality by $\bar{E}'$. This ceiling is again symmetric across m and m', and it is instead decreasing in the diversion ratios D_{12} and D_{21}. Since $\bar{E} \leq \bar{E}'$, we also have the implication

$$\frac{d\hat{y}_m}{dc_m} \leq \frac{dy_m}{dc_m} \Rightarrow \frac{d\hat{y}_{m'}}{dc_m} \geq \frac{dy_{m'}}{dc_m}. \tag{3.29}$$

In addition to the case without synergies, a useful benchmark is the case in which the merged firm attains exactly the synergies needed to keep prices unchanged after the merger. These synergies, first characterized by Werden (1996), satisfy

$$E_m^W := \left(\frac{D_{mm'}}{c_m}\right)\frac{p_{m'} - c_{m'} + D_{m'm}\left(p_m - c_m\right)}{1 - D_{12}D_{21}}. \tag{3.30}$$

The values E_m^W and $E_{m'}^W$ play a crucial role in antitrust policy, because any merger that features smaller synergies will lead to price increases for consumers and hence be deemed anticompetitive. It is then informative to determine when these "Werden synergies" fall above or below the ceilings $\bar{E}$ and $\bar{E}'$.

One useful bound can be formulated in terms of good m's relative markup $\mu_m := p_m/c_m$. In particular, we immediately have

$$E_m^W \geq \frac{D_{12}D_{21}}{1 - D_{12}D_{21}}\left(\mu_m - 1\right). \tag{3.31}$$

Hence $E_m^W \geq \bar{E}$ provided that

$$\mu_m - 1 \geq \frac{1}{4}\frac{1 - D_{12}D_{21}}{1 - \frac{D_{12}D_{21}}{4}}. \tag{3.32}$$

The right side of this inequality is bounded above by $1/4$ for $D_{12}, D_{21} \in [0,1]$. Thus for any merger that achieves Werden synergies, y_m will be less sensitive to c_m if the pre-merger relative markup on m is greater than 25%. A tighter bound can be achieved when goods m and m' are assumed symmetric: $D = D_{mm'} = D_{m'm}$, $p = p_m = p_{m'}$, and $c = c_m = c_{m'}$. Then

$$E_m^W = \frac{D}{1-D}\left(\mu - 1\right) \quad \text{and} \quad \bar{E} = \frac{1}{4}\frac{D^2}{1 - \frac{D^2}{4}}. \tag{3.33}$$

Hence

$$E_m^W \geq \bar{E} \iff \mu - 1 \geq \frac{D\left(1-D\right)}{4 - D^2}. \tag{3.34}$$

The right side of this inequality is bounded above by $\left(2 - \sqrt{3}\right)/4 \approx 0.06699$, so pre-merger relative markups greater than 6.7% imply that y_m will be less sensitive to c_m after a Werden-efficient merger.