# Optimal Unemployment Insurance

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These notes describe the basic theory of optimal unemployment insurance as developed by Baily (1978) and Chetty (2006). This framework emphasizes a fundamental trade-off between insurance and moral hazard: Insurance aids consumption smoothing after job loss, but it also reduces an agent's incentive to exert costly effort to avoid job loss or find new employment. I focus on deriving conditions that characterize optimal unemployment insurance programs under different assumptions on observability of the agent's actions and the structure of equilibrium wage-setting.

## 1 Static Model, Representative Agent, Partial Equilibrium

The simplest model in which the insurance-moral hazard trade-off is salient consists of a representative agent who must exert search effort to become employed. In particular, the agent chooses her *search effort* $e \in [0,1]$ to influence her *employment probability* $\eta : [0,1] \to [0,1]$ subject to an additive utility cost $\psi : [0,1] \to \mathbb{R}_+$. I assume that $\eta$ is twice continuously differentiable, strictly increasing, and weakly concave, with boundary conditions $\eta(0) = 0$ and $\eta(1) = 1$. Similarly, $\psi$ is twice continuously differentiable, strictly increasing, and strictly convex, with boundary conditions

$$\psi(0) = \psi'(0) = 0 \quad \text{and} \quad \lim_{e \uparrow 1} \psi'(e) = \infty. \tag{1.1}$$

If the agent becomes employed, she earns an exogenous wage $w \in \mathbb{R}_{++}$. Otherwise, the agent earns no income. The government seeks to insure the agent against this employment risk, so it implements an *unemployment insurance program* $(c_e, c_u) \in \mathbb{R}^2_+$, an allocation of consumption to the agent when she is employed $c_e$ and when she is unemployed $c_u$.[1] The UI program $(c_e, c_u)$ is constrained to yield weakly positive revenue to the government:

$$\eta(w - c_e) - (1 - \eta) c_u \geq 0. \tag{1.2}$$

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[1] Equivalently, the government chooses a tax $t = w - c_e$ and an unemployment benefit $b = c_u$.

Given a UI program $(c_e, c_u)$, the agent chooses her search effort $e$ to maximize her expected utility:

$$V(c_e, c_u) := \max_{e\in[0,1]} \eta(e)\,v(c_e) + (1-\eta(e))\,u(c_u) - \psi(e). \tag{1.3}$$

Here $v, u : \mathbb{R}_+ \to \bar{\mathbb{R}}$ are the agent's utility functions over consumption when employed and unemployed, respectively.[2] I assume that both $v$ and $u$ are strictly increasing, weakly concave, and continuously differentiable on $\mathbb{R}_+$. In addition, I assume

$$v(w) > u(0) \quad \text{and} \quad v'(w) < u'(0). \tag{1.4}$$

The first inequality ensures that the agent has some incentive to search, while the latter ensures that the agent prefers to shift consumption from the employed state to the unemployed state (i.e., the agent places a positive value on insurance). Since $\eta$ and $\psi$ are continuous, The Extreme Value Theorem implies the existence of a unique solution $e_*(c_e, c_u)\ \forall (c_e, c_u) \in \mathbb{R}_+$. Moreover, since $\eta$ is weakly concave and $\psi$ is strictly convex, when $v(c_e), u(c_u) \in \mathbb{R}$ the following first order condition is necessary and sufficient:

$$0 \geq \eta'(e)[v(c_e) - u(c_u)] - \psi'(e) \quad \text{and} \quad 0 = e\left\{\eta'(e)[v(c_e) - u(c_u)] - \psi'(e)\right\}. \tag{1.5}$$

If $v(c_e) > u(c_u)$, then the solution is necessarily interior, $e_*(c_e, c_u) \in (0,1)$, with

$$0 = \eta'(e_*)[v(c_e) - u(c_u)] - \psi'(e_*). \tag{1.6}$$

## 1.1 Efficiency Problem

In the *efficient* case, the government can observe the agent's search effort $e$. As a result, the government can directly choose $e$ and must solve the problem

$$\begin{aligned} &\max_{e\in[0,1], c_e\geq 0, c_u \geq 0} \eta(e)\,v(c_e) + (1-\eta(e))\,u(c_u) - \psi(e) \qquad (1.7)\\ &\text{subject to}\\ &\eta(e)(w - c_e) - (1-\eta(e))\,c_u \geq 0. \end{aligned}$$

Since $v$ and $u$ are both strictly increasing, the resource constraint will bind at the optimum, and the boundary conditions (1.1) as well as the inequalities (1.4) imply that all choice variables

[2] This specification permits disutility of labor as well as arbitrary complementaries between consumption and labor.

are interior. Letting $\mu > 0$ denote the multiplier on the resource constraint, we have the first order conditions

$$(e) \quad 0 = \eta'(e)[v(c_e) - u(c_u)] - \psi'(e) + \mu\eta'(e)(w - c_e + c_u), \tag{1.8}$$

$$(c_e) \quad 0 = v'(c_e) - \mu, \tag{1.9}$$

$$(c_u) \quad 0 = u'(c_u) - \mu. \tag{1.10}$$

We can derive two key economic insights from these conditions:

(i) (1.9, 1.10) imply that the first-best problem equalizes marginal utilities across employment states. For example, when utility is state-independent and strictly concave, we must have that $c_e = c_u$, so consumption is state-independent.

(ii) (1.8) implies that the first-best choice of $e$ is generally not consistent with the agent's optimality condition (1.5), and in particular with (1.6) since $e$ is interior. With (1.8), (1.6) implies $c_u = c_e - w$. Substituting this equation into the binding resource constraint yields $-c_u \geq 0$, a contradiction since $c_u > 0$ at the optimum. As a result, the first-best problem is not generally implementable when the agent can freely choose her search effort according to (1.3). This intuitively holds because the first-best problem provides positive insurance against the risk of unemployment ($c_u > 0$), diminishing the agent's incentive to exert search effort. This is most apparent in the case with state-independent utility, where optimality implies $v(c_e) = u(c_u)$, and the agent's optimality condition (1.5) immediately yields $e_*(c_e, c_u) = 0$.

## 1.2 Constrained Efficiency Problem

Given the implementation issue with the efficiency problem described above, I turn to an analysis of the *constrained efficient* case in which the government cannot observe the agent's search effort. The government must then choose $c_e, c_u$ to maximize the agent's value function (1.3) subject to the resource constraint (1.2), allowing the agent to privately maximize over $e$:

$$\max_{c_e, c_u} V(c_e, c_u) \tag{1.11}$$

$$\text{subject to}$$

$$\eta_*(w - c_e) - (1 - \eta_*)c_u \geq 0.$$

Here I have written $\eta_* := \eta(e_*(c_e, c_u))$ for notational convenience. I will provide two derivations of optimality conditions for the second-best problem (1.11). The first is the simplest and most common in the literature, making heavy use of the binding budget constraint to define an implicit function; the second follows the Lagrangian approach to optimization. Each method delivers a slightly different optimality condition, though both provide similar insights into the insurance-moral hazard trade-off and generalize to more complex environments.

### 1.2.1 First Approach: Implicit Function

To derive optimality conditions for (1.11), note that our primitive assumptions imply that at the optimum, $c_e, c_u, e_*(c_e, c_u) > 0$. In particular, the solution $e_*$ to the agent's problem (1.3) satisfies the interior first order condition (1.6), and $V$ admits the envelope conditions

$$\frac{\partial V}{\partial c_e} = \eta_* v'(c_e), \tag{1.12}$$
$$\frac{\partial V}{\partial c_u} = (1-\eta_*) u'(c_u). \tag{1.13}$$

In addition, the resource constraint must bind at the optimum, so the Implicit Function Theorem implies that it can be used to define $c_e$ locally as a continuously differentiable function of $c_u$.[3] Differentiating both sides of the resource constraint with respect to $c_u$ then implies that the resource-constant derivative of $c_e$ satisfies

$$\left.\frac{dc_e}{dc_u}\right|_R = -\frac{1-\eta_* - (w-c_e+c_u)\frac{\partial \eta_*}{\partial c_u}}{\eta_* - (w-c_e+c_u)\frac{\partial \eta_*}{\partial c_e}}. \tag{1.14}$$

If we denote the resource-constant derivative of $\eta_*$ with respect to $c_u$ by

$$\left.\frac{d\eta_*}{dc_u}\right|_R := \frac{\partial \eta_*}{\partial c_u} + \frac{\partial \eta_*}{\partial c_e}\left.\frac{dc_e}{dc_u}\right|_R, \tag{1.15}$$

then (1.14) can also be written

$$\left.\frac{dc_e}{dc_u}\right|_R = \frac{1}{\eta_*}\left[(w-c_e+c_u)\left.\frac{d\eta_*}{dc_u}\right|_R - (1-\eta_*)\right] \tag{1.16}$$

[3] The hypotheses of the Theorem are satisfied except for the case in which

$$0 = \frac{\partial}{\partial c_e}\left[\eta_*(w-c_e+c_u)\right] = (w-c_e+c_u)\frac{\partial \eta_*}{\partial c_e} - \eta_*$$

at the optimum. This equality implies that a marginal increase in $c_e$ has no effect on the government's resources at the optimum, which is clearly inconsistent with optimality.

Using this expression, the first order condition for (1.11) with respect to $c_u$ is

$$0 = \left.\frac{dV}{dc_u}\right|_R \tag{1.17}$$

$$= \frac{\partial V}{\partial c_u} + \left.\frac{\partial V}{\partial c_e}\frac{dc_e}{dc_u}\right|_R = (1-\eta_*)u'(c_u) + v'(c_e)\left[(w - c_e + c_u)\left.\frac{d\eta_*}{dc_u}\right|_R - (1-\eta_*)\right]. \tag{1.18}$$

Rearranging, we find

$$\frac{u'(c_u) - v'(c_e)}{v'(c_e)} = -\frac{w - c_e + c_u}{1-\eta_*}\left.\frac{d\eta_*}{dc_u}\right|_R = -\frac{c_u}{\eta_*(1-\eta_*)}\left.\frac{d\eta_*}{dc_u}\right|_R. \tag{1.19}$$

The second equality makes use of the binding resouce constraint. If we define the revenue-constant total elasticity of $1-\eta_*$ with respect to $c_u$

$$\xi_{1-\eta,c_u} := -\frac{c_u}{1-\eta_*}\left.\frac{d\eta_*}{dc_u}\right|_R, \tag{1.20}$$

then we find a version of the *Baily-Chetty Formula*

$$\frac{u'(c_u) - v'(c_e)}{v'(c_e)} = \frac{\xi_{1-\eta,c_u}}{\eta_*}. \tag{1.21}$$

The left side of this equation measures the gap in marginal utilities across employment states and thus the extent to which the insurance provided by the second-best problem deviates from the first-best benchmark. The right side measures the agent's behavioral responses to the unemployment benefit $c_u$ and the revenue-compensating tax $t$ at the optimum, quantifying the moral hazard friction absent in the first-best case. As we will see below, this elasticity is positive for reasonable preferences, so the equation loosely suggests that higher behavioral responses to the UI program correspond to a larger gap in marginal utilities (i.e., less insurance).

In principle, the total elasticity $\xi_{1-\eta,c_u}$ could be directly measured given the correct exogenous variation in the UI program – a small change in $c_u$ with a revenue-compensating change in the tax $t$. This may be infeasible in practice, so it is helpful to understand how $\xi_{1-\eta,c_u}$ can be computed in terms of more primitive "behavioral elasticities." To this end, define the tax and unemployment benefit elasticities

$$\varepsilon_{1-\eta,t} := \frac{w - c_e}{1-\eta_*}\frac{\partial \eta_*}{\partial c_e}, \tag{1.22}$$

$$\varepsilon_{1-\eta,c_u} := -\frac{c_u}{1-\eta_*}\frac{\partial \eta_*}{\partial c_u}, \tag{1.23}$$

Using (1.14, 1.16) as well as the binding resource constraint, simple algebra yields the expression

$$\frac{\xi_{1-\eta,c_u}}{\eta_*} = -1 - \frac{\eta_*}{1-\eta_*}\left.\frac{dc_e}{dc_u}\right|_R = -\frac{-(w-c_e+c_u)\frac{\partial\eta_*}{\partial c_e} + \eta_*\frac{w-c_e+c_u}{1-\eta_*}\frac{\partial\eta_*}{\partial c_u}}{\eta_* - (w-c_e+c_u)\frac{\partial\eta_*}{\partial c_e}} = \frac{\varepsilon_{1-\eta,t} + \varepsilon_{1-\eta,c_u}}{\eta_* - \varepsilon_{1-\eta,t}}. \quad (1.24)$$

In fact, it is straightforward to prove simpler versions of the Baily-Chetty Formula (1.21) that make use of only one of the two behavioral elasticities (1.22, 1.23) instead of the two that are embedded in the total elasticity (1.20). To see this, note that by (1.14), the first order condition (1.18) can be written

$$0 = (1-\eta_*)u'(c_u) - \eta_* v'(c_e)\frac{1-\eta_* - (w-c_e+c_u)\frac{\partial\eta_*}{\partial c_u}}{\eta_* - (w-c_e+c_u)\frac{\partial\eta_*}{\partial c_e}}.$$

Multipliying through by the denominator of the fraction and rearranging, we have

$$\eta_*(1-\eta_*)\left[u'(c_u) - v'(c_e)\right] = (w-c_e+c_u)\left[(1-\eta_*)u'(c_u)\frac{\partial\eta_*}{\partial c_e} - \eta_* v'(c_e)\frac{\partial\eta_*}{\partial c_u}\right]. \quad (1.25)$$

To simplify the right side, we can implicitly differentiate the agent's interior first order condition (1.6) with respect to $c_e$ and $c_u$ to find the expressions

$$\frac{\partial\eta_*}{\partial c_e} = \eta_*'\frac{\partial e_*}{\partial c_e} = -\eta_*'\frac{v'(c_e)}{\eta_*''[v(c_e)-u(c_u)] - \psi''(e_*)},$$
$$\frac{\partial\eta_*}{\partial c_u} = \eta_*'\frac{\partial e_*}{\partial c_u} = \eta_*'\frac{u'(c_u)}{\eta_*''[v(c_e)-u(c_u)] - \psi''(e_*)}.$$

These equations imply the relation

$$\frac{\partial\eta_*}{\partial c_e} = -\frac{v'(c_e)}{u'(c_u)}\frac{\partial\eta_*}{\partial c_u}.$$

Substituting this equation into (1.25) and making use of the binding budget constraint, we can conclude that the following Baily-Chetty variant holds:

$$\frac{u'(c_u) - v'(c_e)}{v'(c_e)} = \frac{\varepsilon_{1-\eta,c_u}}{\eta_*^2}. \quad (1.26)$$

This condition describes the same insurance-moral hazard trade-off as equation (1.21), and it additionally demonstrates that only a single behavioral elasticity $\varepsilon_{1-\eta,c_u}$ is needed to evaluate the moral hazard cost of unemployment insurance, not the "total elasticity" $\xi_{1-\eta,c_u}$ that appears in (1.21) or the two behavioral elasticities that appear in (1.24). In particular, with (1.21), we obtain the following relationship:

$$\xi_{1-\eta,c_u} = \frac{\varepsilon_{1-\eta,c_u}}{\eta_*}. \tag{1.27}$$

Hence at the optimum, the total elasticity $\xi_{1-\eta,c_u}$ is equal to the behavioral elasticity $\varepsilon_{1-\eta,c_u}$, but scaled up by the factor $1/\eta_*$.

Finally, essentially the same argument use to prove (1.26) can be used to prove the equivalent expression

$$\frac{u'(c_u) - v'(c_e)}{u'(c_u)} = \frac{\varepsilon_{1-\eta,t}}{\eta_*(1-\eta_*)}. \tag{1.28}$$

### 1.2.2 Second Approach: Lagrangian Optimization

To begin with the second approach to deriving optimality conditions, let $\mu > 0$ denote the Lagrange multiplier on the government's resource constraint. The (interior) first order conditions with respect to $c_e$ and $c_u$ are then

$$(c_e) \quad 0 = v'(c_e) + \mu\left[\frac{1}{\eta_*}\frac{\partial \eta_*}{\partial c_e}(w - c_e + c_u) - 1\right], \tag{1.29}$$

$$(c_u) \quad 0 = u'(c_u) + \mu\left[\frac{1}{1-\eta_*}\frac{\partial \eta_*}{\partial c_u}(w - c_e + c_u) - 1\right]. \tag{1.30}$$

Subtracting (1.29) from (1.30) and dividing through by $\mu$ give the Baily-Chetty variants

$$\frac{u'(c_u) - v'(c_e)}{\mu} = \left(\frac{1}{\eta_*}\frac{\partial \eta_*}{\partial c_e} - \frac{1}{1-\eta_*}\frac{\partial \eta_*}{\partial c_u}\right)(w - c_e + c_u) \tag{1.31}$$

$$= \frac{\varepsilon_{1-\eta,t} + \varepsilon_{1-\eta,c_u}}{\eta_*} \tag{1.32}$$

The second line follows from the binding resource constraint. For intuition about the first equation, note that (1.14) can be rearranged to give

$$\frac{1}{\eta_*}\frac{\partial \eta_*}{\partial c_e} - \frac{1}{1-\eta_*}\frac{\partial \eta_*}{\partial c_u} = \frac{(w - c_e + c_u)\frac{\partial \eta_*}{\partial c_e} - \eta_*}{\eta_*(1-\eta_*)}\left.\frac{d\eta_*}{dc_u}\right|_R = \frac{\frac{\partial R}{\partial c_e}}{\eta_*(1-\eta_*)}\left.\frac{d\eta_*}{dc_u}\right|_R. \tag{1.33}$$

Here $R(c_e, c_u)$ denotes the revenues collected by the government as a function of the UI system, allowing search effort to vary according to the agent's solution function $e_*$. With this expression, it is easy to see that (1.31) quantifies the optimal trade-off between insurance provision and moral hazard like the standard Baily-Chetty Formula (1.21), but using a more symmetric money metric basis for the gap in marginal utilities.

To summarize, the analysis of this section provided an interpretable optimality condition (one of 1.21, 1.26, 1.28, 1.31, 1.32) that describes how to resolve the insurance-moral hazard trade-off when designing an optimal UI program. We will see below that the analysis generalizes appropriately to richer environments, e.g. with a wage determined in equilibrium or with heterogeneous agents.

# 2 Static Model, Representative Agent, Search Equilibrium

The model presented in Section 1 provides a crucial insight into the nature of unemployment insurance: The government is unable to monitor an agent's search effort, and as a result it must trade off the value of insurance against the disincentive effect. Moreover, the second-best analysis of Section 1.2 demonstrates how the government can resolve this trade-off optimally, culminating in one of the equivalent Baily-Chetty Formulas. However, we may find this model lacking to the extent that the wage is kept fixed, while real unemployment insurance programs have substantive effects on labor market equilibria. To incorporate these macroeconomic concerns, in this section I enrich the model by embedding the agent's problem (1.3) in a general search equilibrium.[4] This exercise is similar to that undertaken by Landais, Michaillat, and Saez (2018).

## 2.1 Search Equilibrium

The economy of Section 1 is modified as follows: There is now a measure one continuum of homogeneous agents searching for jobs. The representative agent chooses her search effort $e$ to influence her employment probability $\eta$ as before, but $\eta : [0,1]^2 \times \mathbb{R}_+ \to [0,1]$ is now also a function $\eta(e,\bar{e},\theta)$ of the *aggregate (mean) search effort* $\bar{e}$ of all agents as well as the *tightness* $\theta$ of the labor market, to be thought of as the "ratio" of vacant firms to searching agents. I assume that $\eta$ is twice continuously differentiable, strictly increasing in $(e,-\bar{e},\theta)$, and weakly concave in $e$, with boundary conditions

$$0 = \eta(0,\bar{e},\theta) = \eta(e,\bar{e},0) \quad \forall e,\bar{e},\theta > 0, \tag{2.1}$$

$$1 = \lim_{\theta \uparrow \infty} \eta(e,\bar{e},\theta) \quad \forall e,\bar{e} > 0. \tag{2.2}$$

[4] Pissaridies (2000) provides a helpful introduction to search equilibria.

By analogy with (1.3), given aggregate search effort $\bar{e}$, tightness $\theta$, and a UI program $(c_e, c_u)$, the representative agent chooses her search effort $e$ to maximize her expected utility:

$$V(\bar{e}, \theta, c_e, c_u,) := \max_{e \in [0,1]} \eta(e, \bar{e}, \theta) v(c_e) + (1 - \eta(e, \bar{e}, \theta)) u(c_u) - \psi(e). \tag{2.3}$$

The necessary and sufficient first order condition is

$$0 \geq \eta_e(e, \bar{e}, \theta)[v(c_e) - u(c_u)] - \psi'(e) \quad \text{and} \quad 0 = e\left\{\eta_e(e, \bar{e}, \theta)[v(c_e) - u(c_u)] - \psi'(e)\right\}. \tag{2.4}$$

If $v(c_e) > u(c_u)$ and $\bar{e} > 0$, then the solution is necessarily interior, $e_*(\bar{e}, c_e, c_u) \in (0, 1)$, with

$$0 = \eta_e(e_*, \bar{e}, \theta)[v(c_e) - u(c_u)] - \psi'(e_*). \tag{2.5}$$

Tightness $\theta : [0, 1] \times \mathbb{R}_+ \to \mathbb{R}_+$ is itself a function $\theta(\bar{e}, w)$ of aggregate search effort $\bar{e}$ and the wage $w$, meant to capture the unmodeled firms' labor demand behavior. I assume that $\theta$ is twice continuously differentiable and strictly increasing in $(\bar{e}, -w)$. Finally, the wage $w : [0, 1] \times \mathbb{R}^3_+ \to \mathbb{R}+$ is a function $w(\bar{e}, \theta, c_e, c_u)$ of aggregate search effort $\bar{e}$, tightness $\theta$, and the unemployment insurance program $(c_e, c_u)$, assumed twice continuously differentiable in all arguments. A (symmetric) equilibrium in this economy is defined as follows:

**Definition 1.** Given a UI program $(c_e, c_u)$, an *equilibrium* is a tuple $(e_*, \theta_*, w_*)$ such that $\theta_* = \theta(e_*, w_*)$, $w_* = w(e_*, \theta_*, c_e, c_u)$, and $e_*$ solves (2.3) given $(\bar{e}, \theta) = (e_*, \theta_*)$.

I waive a discussion of existence and uniqueness of equilibria to focus on optimal UI program design in this economy.

## 2.2 Efficiency Problem

As in Section 1, I begin by considering the *efficiency* problem in which the government can control agents' search effort $e$ and the wage $w$, but must take as given that tightness is set according to the function $\theta(\bar{e}, w)$. This problem allows the government control over all economic variables except the unmodeled firms' labor demand, a natural starting point before we endogenize the wage and search effort.[5]

[5]This is also the simplest problem that remains non-degenerate: If the government could also directly control $\theta$, then it would face no meaningful trade-off when choosing the wage and optimally take $w \uparrow \infty$.

In this case, the government must solve the problem[6]

$$\max_{w\geq 0, e\in[0,1], c_e\geq 0, c_u\geq 0} \eta(e,e,\theta(e,w))\,v(c_e)+(1-\eta(e,e,\theta(e,w)))\,u(c_u)-\psi(e) \tag{2.6}$$

subject to

$$\eta(e,e,\theta(e,w))(w-c_e)-(1-\eta(e,e,\theta(e,w)))\,c_u\geq 0.$$

The resource constraint must bind at the optimum, and I will assume the choice variables and the equilibrium variables take interior values. Letting $\mu>0$ denote the multiplier on the resource constraint, the first order conditions are

$$(w)\quad 0=\eta_\theta\theta_w[v(c_e)-u(c_u)]+\mu\{\eta+\eta_\theta\theta_w(w-c_e+c_u)\}, \tag{2.7}$$

$$(e)\quad 0=(\eta_e+\eta_{\bar{e}}+\eta_\theta\theta_{\bar{e}})[v(c_e)-u(c_u)]-\psi'(e)+\mu(\eta_e+\eta_{\bar{e}}+\eta_\theta\theta_{\bar{e}})(w-c_e+c_u), \tag{2.8}$$

$$(c_e)\quad 0=v'(c_e)-\mu, \tag{2.9}$$

$$(c_u)\quad 0=u'(c_u)-\mu. \tag{2.10}$$

A few observations:

(i) (2.9, 2.10) imply that the half-best problem equalizes marginal utilities across employment states, just as with the first-best problem (1.7) from Section 1.1. As before, this implies that consumption is state-independent when utility is state-independent and strictly concave. Intuitively, this property of the solution persists with endogenous tightness because tightness $\theta(\bar{e},w)$ is not directly responsive to the UI program $(c_e,c_u)$.

(ii) (2.8) implies that the first-best choice of $e$ is generally not consistent with the agent's optimality condition (2.5). By contrast with the first-best problem (1.7) from Section 1.1, this inconsistency arises for two reasons. As before, the first-best problem provides insurance against unemployment risk to such an extent that the representative agent would not find it optimal to exert the first-best level of effort $e$. However, in this environment there is an additional reason why the first-best effort choice is inconsistent with agent optimality. To see this, note that (2.8) can be written

$$\begin{aligned}0=&\ \eta_e[v(c_e)-u(c_u)]-\psi'(e)+\mu\eta_e(w-c_e+c_u)\\&+(\eta_{\bar{e}}+\eta_\theta\theta_{\bar{e}})\{[v(c_e)-u(c_u)]+\mu(w-c_e+c_u)\}.\end{aligned} \tag{2.11}$$

[6] Note that in this problem, the government chooses both individual search effort $e$ and aggregate search effort $\bar{e}=e$. I will generally assume in my analysis that the government can choose an equilibrium when multiplicity obtains.

Comparing with (1.8), we find that the government must now take into account *aggregate effort externalities* on both the representative agent's expected utility as well as the government's own budget constraint, given by the expression on the second line of (2.11). These externalities arise because tightness $\theta$ and the representative agent's match probability $\eta$ are both directly affected by aggregate search effort $\bar{e}$, which the agent takes as given when determining her individual search effort $e$.

(iii) (2.7) provides a definition for a *socially efficient wage*, i.e., one that equalizes the marginal effect on the agent's expected utility with the value of the marginal change in the government's budget constraint.

## 2.3 Partially Constrained $(e)$ Problem

In the *PCE*$(e)$ case, the government can set the wage $w$ arbitrarily, but it can no longer monitor the agent's search effort $e$. As a result, the government solves the problem

$$\max_{w\geq 0, c_e \geq 0, c_u \geq 0} \eta(e_*, e_*, \theta_*) v(c_e) + (1-\eta(e_*, e_*, \theta_*)) u(c_u) - \psi(e_*) \tag{2.12}$$

subject to

$$\eta(e_*, e_*, \theta_*)(w - c_e) - (1-\eta(e_*, e_*, \theta_*)) c_u \geq 0.$$

Here $\theta_*$, $e_*$ satisfy the equilibrium system

$$\theta_* = \theta(e_*, w), \tag{2.13}$$

$$0 = \eta_e(e_*, e_*, \theta_*)[v(c_e) - u(c_u)] - \psi'(e_*). \tag{2.14}$$

Substituting (2.13) into (2.14), we can reduce the equilibrium system to

$$0 = \eta_e(e_*, e_*, \theta(e_*, w))[v(c_e) - u(c_u)] - \psi'(e_*). \tag{2.15}$$

I assume that the Implicit Function Theorem suffices to imply that $\theta_*, e_*$ are locally continuously differentiable functions of the primitives $w, c_e, c_u$.

As in Section 1.2, I derive optimality conditions using both the implicit function and Lagrangian approaches.

### 2.3.1 First Approach: Implicit Function

Similarly to before, in the first approach I use the binding resource constraint to define $c_e$ as a locally continuously differentiable function of $c_u$ and $w$. I assume that the Implicit Function Theorem applies, in which case the revenue-constant total derivative of $c_e$ with respect to $c_u$, holding $w$ constant, is

$$\left.\frac{dc_e}{dc_u}\right|_{R,w} := -\frac{(\eta_e + \eta_{\bar{e}} + \eta_\theta \theta_{\bar{e}})(w - c_e + c_u)\frac{\partial e_*}{\partial c_u} - (1-\eta)}{(\eta_e + \eta_{\bar{e}} + \eta_\theta \theta_{\bar{e}})(w - c_e + c_u)\frac{\partial e_*}{\partial c_e} - \eta}. \tag{2.16}$$

If we denote the resource-constant derivative of $\eta$ with respect to $c_u$ by

$$\left.\frac{d\eta}{dc_u}\right|_{R,w} := (\eta_e + \eta_{\bar{e}} + \eta_\theta \theta_{\bar{e}})\left(\frac{\partial e_*}{\partial c_u} + \frac{\partial e_*}{\partial c_e}\left.\frac{dc_e}{dc_u}\right|_{R,w}\right), \tag{2.17}$$

then (2.16) can be written

$$\left.\frac{dc_e}{dc_u}\right|_{R,w} = \frac{1}{\eta}\left[(w - c_e + c_u)\left.\frac{d\eta}{dc_u}\right|_{R,w} - (1-\eta)\right]. \tag{2.18}$$

Using this expression, we can differentiate the objective of the PCE($e$) problem (2.12) while holding revenue and $w$ constant to find the first order condition

$$0 = v'(c_e)\left[(w - c_e + c_u)\left.\frac{d\eta}{dc_u}\right|_{R,w} - (1-\eta)\right] + (1-\eta)u'(c_u) \tag{2.19}$$

$$+ (\eta_{\bar{e}} + \eta_\theta \theta_{\bar{e}})\left.\frac{\partial e}{\partial c_u}\right|_{R,w}(v(c_e) - u(c_u)) \tag{2.20}$$

Rearranging,

$$\frac{u'(c_u) - v'(c_e)}{v'(c_e)} = -\frac{w - c_e + c_u}{1-\eta}\left.\frac{d\eta}{dc_u}\right|_{R,w} - \frac{\eta_{\bar{e}} + \eta_\theta \theta_{\bar{e}}}{1-\eta}\left.\frac{\partial e}{\partial c_u}\right|_{R,w}\frac{v(c_e) - u(c_u)}{v'(c_e)}. \tag{2.21}$$

Define the revenue- and wage-constant total elasticities

$$\left.\xi_{1-\eta,c_u}\right|_{R,w} := -\frac{c_u}{1-\eta}\left.\frac{d\eta}{dc_u}\right|_{R,w}, \tag{2.22}$$

$$\left.\xi_{1-\eta,c_u}\right|_{R,w}^{\text{micr}} := -\frac{c_u}{1-\eta}\eta_e\left.\frac{\partial e}{\partial c_u}\right|_{R,w}, \tag{2.23}$$

$$\left.\xi_{1-\eta,c_u}\right|_{R,w}^{\text{ext}} := -\frac{c_u}{1-\eta}(\eta_{\bar{e}} + \eta_\theta \theta_{\bar{e}})\left.\frac{\partial e}{\partial c_u}\right|_{R,w}. \tag{2.24}$$

Note that the second elasticity measures the response of $1-\eta$ holding changes in aggregate search effort $\bar{e}$ constant, while the third elasticity measures the response of $1-\eta$ holding constant the individual agent's search effort. Then (2.21) implies the Baily-Chetty variants

$$\frac{u'(c_u)-v'(c_e)}{v'(c_e)} = \frac{\xi_{1-\eta,c_u}\big|_{R,w}}{\eta} + \frac{1}{c_u}\,\xi_{1-\eta,c_u}\big|_{R,w}^{\text{ext}}\,\frac{v(c_e)-u(c_u)}{v'(c_e)}, \tag{2.25}$$

$$= \frac{\xi_{1-\eta,c_u}\big|_{R,w}^{\text{micr}}}{\eta} + \left[\frac{v(c_e)-u(c_u)}{v'(c_e)} + (w-c_e+c_u)\right]\frac{1}{c_u}\,\xi_{1-\eta,c_u}\big|_{R,w}^{\text{ext}} \tag{2.26}$$

Equation (2.26) is particularly helpful for developing intuition about the solution: The government trades off the value of insurance (left side) against the moral hazard cost from the representative agent (first term on the right side) and the value of a change in the unemployment probability due to aggregate effort externalities (second term on the right side). Notably, the government deviates from the standard Baily-Chetty provision of insurance for the purpose of correcting the aggregate effort externalities, which cannot be corrected through a Pigouvian tax due to asymmetric information about search effort.

### 2.3.2 Second Approach: Lagrangian Optimization

For the Lagrangian approach, let $\mu > 0$ denote the multiplier on the resource constraint. The first order conditions for the PCE($e$) problem (2.12) are

$$0 = \left[(\eta_{\bar{e}} + \eta_\theta \theta_{\bar{e}})\frac{\partial e_*}{\partial w} + \eta_\theta \theta_w\right][v(c_e)-u(c_u)] \tag{2.27}$$

$$+\mu\left\{\left[(\eta_e + \eta_{\bar{e}} + \eta_\theta \theta_{\bar{e}})\frac{\partial e_*}{\partial w} + \eta_\theta \theta_w\right](w-c_e+c_u) + \eta\right\}, \tag{2.28}$$

$$0 = \eta v'(c_e) + (\eta_{\bar{e}} + \eta_\theta \theta_{\bar{e}})\frac{\partial e_*}{\partial c_e}[v(c_e)-u(c_u)] \tag{2.29}$$

$$+\mu\left\{(\eta_e + \eta_{\bar{e}} + \eta_\theta \theta_{\bar{e}})\frac{\partial e_*}{\partial c_e}(w-c_e+c_u) - \eta\right\}, \tag{2.30}$$

$$0 = (1-\eta)u'(c_u) + (\eta_{\bar{e}} + \eta_\theta \theta_{\bar{e}})\frac{\partial e_*}{\partial c_u}[v(c_e)-u(c_u)] \tag{2.31}$$

$$+\mu\left\{(\eta_e + \eta_{\bar{e}} + \eta_\theta \theta_{\bar{e}})\frac{\partial e_*}{\partial c_u}(w-c_e+c_u) - (1-\eta)\right\}. \tag{2.32}$$

Divide (2.29, 2.31) respectively by $\eta\mu$ and $(1-\eta)\mu$ and subtract the resulting equations to find the Baily-Chetty variants

$$\frac{u'(c_u)-v'(c_e)}{\mu} = \left[\eta_e(w-c_e+c_u)+(\eta_{\bar{e}}+\eta_\theta\theta_{\bar{e}})\left(\frac{v(c_e)-u(c_u)}{\mu}+(w-c_e+c_u)\right)\right]\left(\frac{1}{\eta}\frac{\partial e_*}{\partial c_e}-\frac{1}{1-\eta}\frac{\partial e_*}{\partial c_u}\right) \tag{2.33}$$

## 2.4 Partially Constrained $(w)$ Efficiency Problem

In the *PCE*($w$) case, the government can still observe the agent's search effort $e$, but it can no longer set the wage $w$ arbitrarily. As a result, the government must solve the problem

$$\max_{e\in[0,1],c_e\geq 0,c_u\geq 0} \eta(e,e,\theta_*)v(c_e)+(1-\eta(e,e,\theta_*))u(c_u)-\psi(e) \tag{2.34}$$

subject to

$$\eta(e,e,\theta_*)(w_*-c_e)-(1-\eta(e,e,\theta_*))c_u \geq 0.$$

With observable effort, the government is allowed discretion over one component $e_*$ of the equilibrium, while the remaining components $\theta_*, w_*$ are considered structural elements of the economy that cannot be directly manipulated. The resource constraint must bind at the optimum, and I will assume the choice variables and the equilibrium variables take interior values. To use differential methods to derive first order conditions, we must first verify that the equilibrium variables $\theta_*, w_*$ change smoothly in response to the choice variables $e, c_e, c_u$. To do this, note that $\theta_*, w_*$ satisfy the nonlinear system

$$\theta_* = \theta(e,w_*), \tag{2.35}$$

$$w_* = w(e,\theta_*,c_e,c_u). \tag{2.36}$$

Substituting (2.35) into (2.36), we can reduce the equilibrium system to

$$w_* = w(e,\theta(e,w_*),c_e,c_u). \tag{2.37}$$

To ensure that this equation implies that $w_*$ is a locally continuously differentiable function of $e, c_e, c_u$, I assume

$$0 \neq \frac{d}{dw_*}\{w_*-w(e,\theta(e,w_*),c_e,c_u)\} = 1-w_\theta\theta_w \iff w_\theta\theta_w \neq 1. \tag{2.38}$$

With this condition satisfied, the Implicit Function Theorem implies that $\theta_*, w_*$ are locally continuously differentiable functions of the primitives $e, c_e, c_u$.[7]

### 2.4.1 First Approach: Implicit Function

In the first approach, I use the binding resource constraint to define $c_e$ as a locally continuously differentiable function of $c_u$ and $e$. The Implicit Function Theorem ensures that this is possible when

$$\begin{aligned} 0 &\neq \frac{d}{dc_e}\{\eta(e,e,\theta_*)(w_*-c_e)-(1-\eta(e,e,\theta_*))c_u\} \\ &= \{\eta+\eta_\theta\theta_w(w_*-c_e+c_u)\}\frac{\partial w_*}{\partial c_e}-\eta. \end{aligned}$$

Assuming that this condition holds, the revenue-constant total derivative of $c_e$ with respect to $c_u$, holding $e$ constant, is then

$$\left.\frac{dc_e}{dc_u}\right|_{R,e} := -\frac{\{\eta+\eta_\theta\theta_w(w_*-c_e+c_u)\}\frac{\partial w_*}{\partial c_u}-(1-\eta)}{\{\eta+\eta_\theta\theta_w(w_*-c_e+c_u)\}\frac{\partial w_*}{\partial c_e}-\eta} \tag{2.39}$$

If we denote the resource-constant derivatives of $w_*$ and $\eta$ with respect to $c_u$ by

$$\left.\frac{dw_*}{dc_u}\right|_{R,e} := \frac{\partial w_*}{\partial c_u}+\frac{\partial w_*}{\partial c_e}\left.\frac{dc_e}{dc_u}\right|_{R,e}, \tag{2.40}$$

$$\left.\frac{d\eta}{dc_u}\right|_{R,e} := \eta_\theta\theta_w\left.\frac{dw_*}{dc_u}\right|_{R,e}, \tag{2.41}$$

then (2.39) can be written

$$\left.\frac{dc_e}{dc_u}\right|_{R,e} = \frac{1}{\eta}\left[(w-c_e+c_u)\left.\frac{d\eta}{dc_u}\right|_{R,e}+\eta\left.\frac{dw_*}{dc_u}\right|_{R,e}-(1-\eta)\right]. \tag{2.42}$$

[7] Note that in what follows, when taking partial derivatives of $\theta$ and $w$, I hold the remaining arguments in the given function constant. When taking partial derivatives of the equilibrium variables $\theta_*$ and $w_*$, I hold the remaining exogenously fixed variables constant. (In this case, $e, c_e, c_u$ are the exogenously fixed variables.)

Using this expression, we can differentiate the objective function of the PCE($w$) problem (2.34) while holding revenue and $e$ constant to find the first order condition

$$
\begin{aligned}
0 = v'(c_e)\left[(w - c_e + c_u)\left.\frac{d\eta}{dc_u}\right|_{R,e} + \eta \left.\frac{dw_*}{dc_u}\right|_{R,e} - (1-\eta)\right] + (1-\eta)u'(c_u) \\
+ \eta_\theta \theta_w \left.\frac{dw_*}{dc_u}\right|_{R,e} [v(c_e) - u(c_u)]
\end{aligned}
\tag{2.43}
$$

Rearranging,

$$
\frac{u'(c_u) - v'(c_e)}{v'(c_e)} = -\frac{w - c_e + c_u}{1-\eta} \left.\frac{d\eta}{dc_u}\right|_{R,e} - \frac{1}{1-\eta}\left(\eta + \eta_\theta \theta_w \frac{v(c_e) - u(c_u)}{v'(c_e)}\right) \left.\frac{dw_*}{dc_u}\right|_{R,e}. \tag{2.44}
$$

By analogy with (1.20), define the revenue- and effort-constant total elasticities of $1-\eta$ and $w_*$ with respect to $c_u$:

$$
\left.\xi_{1-\eta,c_u}\right|_{R,e} := -\frac{c_u}{1-\eta} \left.\frac{d\eta}{dc_u}\right|_{R,e}, \tag{2.45}
$$

$$
\left.\xi_{w,c_u}\right|_{R,e} := \frac{c_u}{w} \left.\frac{dw_*}{dc_u}\right|_{R,e}. \tag{2.46}
$$

Then (2.44) implies the Baily-Chetty variants

$$
\frac{u'(c_u) - v'(c_e)}{v'(c_e)} = \frac{\left.\xi_{1-\eta,c_u}\right|_{R,e}}{\eta} - \frac{1}{1-\eta}\left(\eta + \eta_\theta \theta_w \frac{v(c_e) - u(c_u)}{v'(c_e)}\right) \frac{w_*}{c_u} \left.\xi_{w,c_u}\right|_{R,e} \tag{2.47}
$$

$$
= -\frac{1}{1-\eta}\left[\eta_\theta \theta_w \frac{v(c_e) - u(c_u)}{v'(c_e)} + \eta + \eta_\theta \theta_w (w_* - c_e + c_u)\right] \frac{w_*}{c_u} \left.\xi_{w,c_u}\right|_{R,e} \tag{2.48}
$$

These equations demonstrate that the key trade-off in the PCE($w$) problem relates to the endogeneity of the wage: The government is willing to deviate from full insurance for the purpose of influencing the wage. To see this, note that the bracketed term in (2.48) describes the total value of increasing the wage marginally, in units of consumption while employed. If this term is positive and if increasing $c_u$ in a revenue-constant manner increases the wage, then (2.48) shows that the government will generally *over-insure* the agent.

### 2.4.2 Second Approach: Lagrangian Optimization

To apply the Lagrangian method, let $\mu > 0$ denote the multiplier on the resource constraint. The first order conditions to the PCE($w$) problem (2.34) are

$$\begin{aligned} (e) \quad 0 &= \left(\eta_e + \eta_{\bar{e}} + \eta_\theta \frac{\partial \theta_*}{\partial \bar{e}}\right)\left[v(c_e) - u(c_u)\right] - \psi'(e) \\ &\quad + \mu\left\{\eta \frac{\partial w_*}{\partial \bar{e}} + \left(\eta_e + \eta_{\bar{e}} + \eta_\theta \frac{\partial \theta_*}{\partial \bar{e}}\right)(w_* - c_e + c_u)\right\}, \end{aligned} \tag{2.49}$$

$$\begin{aligned} (c_e) \quad 0 &= \eta v'(c_e) + \eta_\theta \frac{\partial \theta_*}{\partial c_e}\left[v(c_e) - u(c_u)\right] \\ &\quad + \mu\left\{\eta\left(\frac{\partial w_*}{\partial c_e} - 1\right) + \eta_\theta \frac{\partial \theta_*}{\partial c_e}(w_* - c_e + c_u)\right\}, \end{aligned} \tag{2.50}$$

$$\begin{aligned} (c_u) \quad 0 &= (1-\eta)u'(c_u) + \eta_\theta \frac{\partial \theta_*}{\partial c_u}\left[v(c_e) - u(c_u)\right] \\ &\quad + \mu\left\{\eta \frac{\partial w_*}{\partial c_u} - (1-\eta) + \eta_\theta \frac{\partial \theta_*}{\partial c_u}(w_* - c_e + c_u)\right\}. \end{aligned} \tag{2.51}$$

Since $\theta_* = \theta(\bar{e}, w_*)$ in equilibrium, these first order conditions can be written

$$\begin{aligned} (e) \quad 0 &= \frac{1}{\mu}\left\{(\eta_e + \eta_{\bar{e}} + \eta_\theta \theta_{\bar{e}})\left[v(c_e) - u(c_u)\right] - \psi'(e)\right\} + (\eta_e + \eta_{\bar{e}} + \eta_\theta \theta_{\bar{e}})(w_* - c_e + c_u) \\ &\quad + \left\{\eta_\theta \theta_w \frac{v(c_e) - u(c_u)}{\mu} + \eta + \eta_\theta \theta_w (w_* - c_e + c_u)\right\}\frac{\partial w_*}{\partial \bar{e}}, \end{aligned} \tag{2.52}$$

$$(c_e) \quad 0 = \eta\left[\frac{v'(c_e)}{\mu} - 1\right] + \left\{\eta_\theta \theta_w \frac{v(c_e) - u(c_u)}{\mu} + \eta + \eta_\theta \theta_w (w_* - c_e + c_u)\right\}\frac{\partial w_*}{\partial c_e}, \tag{2.53}$$

$$(c_u) \quad 0 = (1-\eta)\left[\frac{u'(c_u)}{\mu} - 1\right] + \left\{\eta_\theta \theta_w \frac{v(c_e) - u(c_u)}{\mu} + \eta + \eta_\theta \theta_w (w_* - c_e + c_u)\right\}\frac{\partial w_*}{\partial c_u}. \tag{2.54}$$

The common structure of these conditions is clear: When choosing one of the variables $e, c_e, c_u$, the government balances the direct welfare and fiscal effects of a marginal change in the choice variable against the indirect welfare and fiscal effects that arise through an adjustment in the wage. The first order condition for a choice variable collapses to the efficient condition when the wage's welfare and fiscal effects sum to zero (i.e., the equilibrium wage is socially efficient) or when the choice variable has no effect on the wage.

As in the efficient case, effort $e$ is generally not consistent with agent optimality (compare (2.52) and (2.5)). In this case, an additional complication arises because the optimal choice of $e$ must generally take into account feedback effects through the wage $w$. More importantly, marginal utilities of consumption are not generally equated across employment states. To see this, respectively divide (2.53) and (2.54) by $\eta$ and $1-\eta$ and subtract the resulting equations to find the Baily-Chetty variants

$$\frac{u'(c_u)-v'(c_e)}{\mu} = \left\{\eta_\theta \theta_w \frac{v(c_e)-u(c_u)}{\mu} + \eta + \eta_\theta \theta_w (w_* - c_e + c_u)\right\}\left(\frac{1}{\eta}\frac{\partial w_*}{\partial c_e} - \frac{1}{1-\eta}\frac{\partial w_*}{\partial c_u}\right) \tag{2.55}$$

$$= \left\{\eta_\theta \theta_w \frac{v(c_e)-u(c_u)}{\mu} + \eta + \eta_\theta \theta_w (w_* - c_e + c_u)\right\}\left(\frac{\varepsilon_{w_*,c_e}}{\eta c_e} - \frac{\varepsilon_{w_*,c_u}}{(1-\eta)c_u}\right) w_*. \tag{2.56}$$

In (2.56), I have used the partial elasticities

$$\varepsilon_{w_*,c_e} := \frac{c_e}{w_*}\frac{\partial w_*}{\partial c_e}, \tag{2.57}$$

$$\varepsilon_{w_*,c_u} := \frac{c_u}{w_*}\frac{\partial w_*}{\partial c_u}. \tag{2.58}$$

For useful intuition about (2.55), note that (2.39) can be rearranged to give

$$\frac{1}{\eta}\frac{\partial w_*}{\partial c_e} - \frac{1}{1-\eta}\frac{\partial w_*}{\partial c_u} = \frac{\{\eta + \eta_\theta \theta_w (w_* - c_e + c_u)\}\frac{\partial w_*}{\partial c_e} - \eta}{\eta(1-\eta)}\left(\frac{\partial w_*}{\partial c_e}\left.\frac{dc_e}{dc_u}\right|_{R,e} + \frac{\partial w_*}{\partial c_u}\right)$$

$$= \frac{\left.\frac{\partial R}{\partial c_e}\right|_{e,c_u}}{\eta(1-\eta)}\left.\frac{dw_*}{dc_e}\right|_{R,e}. \tag{2.59}$$

This calculation implies that the term on the left essentially measures the response of the equilibrium wage $w_*$ to a resource-constant increase in consumption while employed $c_e$. Comparing with (2.55), we again find that equalizing marginal utilities is optimal if and only if the wage is socially optimal at the PCE($w$) solution or if the wage is not responsive to marginal changes in consumption across states that leave total government resources unchanged. Otherwise, the government has an incentive to pay a (second-order) cost of reducing insurance to obtain a (first-order) gain from moving the wage closer to the socially efficient level.

## 2.5 Constrained Efficiency Problem

We now turn to an analysis of the *constrained efficient* case in which the government chooses the UI program $(c_e, c_u)$ to maximize the representative agent's utility, subject to a resource constraint and the constraint that $(e_*, \theta_*, w_*)$ gives an equilibrium:

$$\max_{c_e, c_u} V(e_*, \theta_*, c_e, c_u) \tag{2.60}$$
subject to
$$\eta_*(w_* - c_e) - (1-\eta_*) c_u \geq 0$$
$$(e_*, \theta_*, w_*) \text{ an equilibrium.}$$

As in Section 1.2, I derive optimality conditions using both the implicit function and Lagrangian approaches.

### 2.5.1 First Approach: Implicit Function

Assuming a solution to (2.60) such that $c_e, c_u, e_*, \theta_* > 0$, $V$ admits the envelope conditions

$$\frac{\partial V}{\partial \bar{e}} = \eta_{\bar{e}}\left[v(c_e) - u(c_u)\right], \tag{2.61}$$
$$\frac{\partial V}{\partial \theta} = \eta_{\theta}\left[v(c_e) - u(c_u)\right], \tag{2.62}$$
$$\frac{\partial V}{\partial c_e} = \eta v'(c_e), \tag{2.63}$$
$$\frac{\partial V}{\partial c_u} = (1-\eta) u'(c_u). \tag{2.64}$$

We must also ensure that the equilibrium variables $e_*, \theta_*, w_*$ change smoothly in response to the choice variables $e, c_e, c_u$. Subsituting $\theta_* = \theta(e_*, w_*)$, we have that the variables $e_*, w_*$ satisfy the nonlinear system

$$0 = \eta_e(e_*, e_*, \theta(e_*, w_*))\left[v(c_e) - u(c_u)\right] - \psi'(e_*), \tag{2.65}$$
$$0 = w_* - w(e_*, \theta(e_*, w_*), c_e, c_u). \tag{2.66}$$

To ensure that these equations imply that $e_*, w_*$ are locally continuously differentiable functions of $c_e, c_u$, I assume

$$0 \neq \left\{(\eta_{ee} + \eta_{e\bar{e}} + \eta_{e\theta}\theta_e)\left[v(c_e) - u(c_u)\right] - \psi''(e_*)\right\}\{1 - w_\theta \theta_w\} \tag{2.67}$$
$$+ \{w_{\bar{e}} + w_\theta \theta_{\bar{e}}\} \eta_{e\theta}\theta_w \left[v(c_e) - u(c_u)\right]. \tag{2.68}$$

Now the resource constraint in the constrained efficiency problem (2.60) must bind at the optimum, so we can use the Implicit Function Theorem to define $c_e$ locally as a continuously differentiable function of $c_u$.[8] The resource-constant derivative of $c_e$ with respect to $c_u$ is then

$$\left.\frac{dc_e}{dc_u}\right|_R := -\frac{1-\eta_* - \frac{\partial\eta_*}{\partial c_u}(w_* - c_e + c_u) - \eta_* \frac{\partial w_*}{\partial c_u}}{\eta_* - \frac{\partial\eta_*}{\partial c_e}(w_* - c_e + c_u) - \eta_* \frac{\partial w_*}{\partial c_e}}. \tag{2.69}$$

If we define the revenue-constant derivatives of $e_*, \theta_*, w_*, \eta_*$ with respect to $c_u$ by

$$\left.\frac{de_*}{dc_u}\right|_R := \frac{\partial e_*}{\partial c_e}\left.\frac{dc_e}{dc_u}\right|_R + \frac{\partial e_*}{\partial c_u}, \tag{2.70}$$

$$\left.\frac{d\theta_*}{dc_u}\right|_R := \frac{\partial \theta_*}{\partial c_e}\left.\frac{dc_e}{dc_u}\right|_R + \frac{\partial \theta_*}{\partial c_u}, \tag{2.71}$$

$$\left.\frac{dw_*}{dc_u}\right|_R := \frac{\partial w_*}{\partial c_e}\left.\frac{dc_e}{dc_u}\right|_R + \frac{\partial w_*}{\partial c_u}, \tag{2.72}$$

$$\left.\frac{d\eta_*}{dc_u}\right|_R := (\eta_e + \eta_{\bar{e}} + \eta_\theta \theta_{\bar{e}})\left.\frac{de_*}{dc_u}\right|_R + \eta_\theta \theta_w \left.\frac{dw_*}{dc_u}\right|_R, \tag{2.73}$$

then (2.69) can also be written

$$\left.\frac{dc_e}{dc_u}\right|_R = \frac{1}{\eta}\left\{\left.\frac{d\eta_*}{dc_u}\right|_R (w_* - c_e + c_u) - (1-\eta_*) + \eta_* \left.\frac{dw_*}{dc_u}\right|_R\right\}. \tag{2.74}$$

The first-order condition of (2.60) is then

$$\begin{aligned}
0 &= \left.\frac{dV}{dc_u}\right|_R \\
&= V_{\bar{e}}\left.\frac{de_*}{dc_u}\right|_R + V_\theta \left.\frac{d\theta_*}{dc_u}\right|_R + V_{c_e}\left.\frac{dc_e}{dc_u}\right|_R + V_{c_u} \\
&= \left\{\eta_{\bar{e}}\left.\frac{de_*}{dc_u}\right|_R + \eta_\theta \left.\frac{d\theta_*}{dc_u}\right|_R\right\}[v(c_e) - u(c_u)] \\
&\quad + v'(c_e)\left\{\left.\frac{d\eta_*}{dc_u}\right|_R (w_* - c_e + c_u) - (1-\eta_*) + \eta_* \left.\frac{dw_*}{dc_u}\right|_R\right\} + (1-\eta_*)u'(c_u).
\end{aligned}$$

[8] Again, we must exclude that case in which

$$0 = \frac{\partial}{\partial c_e}\{\eta_*(w_* - c_e + c_u)\} = \eta_*\left(\frac{\partial w_*}{\partial c_e} - 1\right) + (w_* - c_e + c_u)\frac{\partial \eta_*}{\partial c_e}$$

at the optimum, which would be inconsistent with optimality anyway.

Rearranging, we have

$$\begin{aligned}
&(1-\eta_*)\frac{u'(c_u)-v'(c_e)}{v'(c_e)} \\
&=-\left.\frac{d\eta_*}{dc_u}\right|_R (w_*-c_e+c_u)-\eta_*\left.\frac{dw_*}{dc_u}\right|_R-\left\{\eta_{\bar{e}}\left.\frac{de_*}{dc_u}\right|_R+\eta_\theta\left.\frac{d\theta_*}{dc_u}\right|_R\right\}\frac{v(c_e)-u(c_u)}{v'(c_e)}.
\end{aligned} \tag{2.75}$$

To simplify, define the equilibrium total elasticities

$$\left.\xi_{1-\eta,c_u}\right|_R := -\frac{c_u}{1-\eta_*}\left.\frac{d\eta_*}{dc_u}\right|_R, \tag{2.76}$$

$$\left.\xi_{1-\eta,c_u}\right|_R^{\text{micro}} := -\frac{c_u}{1-\eta_*}\eta_e\left.\frac{de_*}{dc_u}\right|_R, \tag{2.77}$$

$$\left.\xi_{1-\eta,c_u}\right|_R^{\text{ext}} := -\frac{c_u}{1-\eta_*}(\eta_{\bar{e}}+\eta_\theta\theta_{\bar{e}})\left.\frac{de_*}{dc_u}\right|_R, \tag{2.78}$$

$$\left.\xi_{w,c_u}\right|_R := \frac{c_u}{w_*}\left.\frac{dw_*}{dc_u}\right|_R. \tag{2.79}$$

To emphasize, these are *total* elasticities in the sense that they include adjustments in $c_e$ to satisfy the resource constraint, and they are *equilibrium* elasticities in the sense that they allow all equilibrium variables $e_*, \theta_*, w_*$ to change according to the equilibrium system (2.65, 2.66).Substituting the elasticities into (2.75) and making use of the binding resource constraint, we find the Baily-Chetty variants

$$\begin{aligned}
&\frac{u'(c_u)-v'(c_e)}{v'(c_e)} \\
&=\frac{\left.\xi_{1-\eta,c_u}\right|_R}{\eta_*}-\frac{\eta_*}{1-\eta_*}\frac{w_*}{c_u}\left.\xi_{w,c_u}\right|_R+\frac{1}{c_u}\left\{\left.\xi_{1-\eta,c_u}\right|_R^{\text{ext}}-\eta_\theta\theta_w w_*\left.\xi_{w,c_u}\right|_R\right\}\frac{v(c_e)-u(c_u)}{v'(c_e)}
\end{aligned} \tag{2.80}$$

$$\begin{aligned}
&=\frac{\left.\xi_{1-\eta,c_u}\right|_R^{\text{micro}}}{\eta_*}-\left\{\eta_\theta\theta_w\frac{v(c_e)-u(c_u)}{v'(c_e)}+\eta_*+\eta_\theta\theta_w(w_*-c_e+c_u)\right\}\frac{w_*}{c_u}\left.\xi_{w,c_u}\right|_R \\
&\quad+\left\{\frac{v(c_e)-u(c_u)}{v'(c_e)}+(w-c_e+c_u)\right\}\frac{1}{c_u}\left.\xi_{1-\eta,c_u}\right|_R^{\text{ext}}
\end{aligned} \tag{2.81}$$

### 2.5.2 Second Approach: Lagrangian Optimization

For the second approach, let $\mu > 0$ denote the multiplier on the resource constraint. The first order conditions for the constrained efficiency problem (2.60) are then

$$
\begin{aligned}
(c_e)\quad 0 &= \eta v'(c_e) + \left[(\eta_{\bar{e}} + \eta_\theta \theta_{\bar{e}})\frac{\partial e_*}{\partial c_e} + \eta_\theta \theta_w \frac{\partial w_*}{\partial c_e}\right][v(c_e) - u(c_u)] \\
&\quad + \mu\left\{\eta \frac{\partial w_*}{\partial c_e} - \eta + \left[(\eta_e + \eta_{\bar{e}} + \eta_\theta \theta_{\bar{e}})\frac{\partial e_*}{\partial c_e} + \eta_\theta \theta_w \frac{\partial w_*}{\partial c_e}\right](w_* - c_e + c_u)\right\}, \qquad (2.82) \\
(c_u)\quad 0 &= (1-\eta)u'(c_u) + \left[(\eta_{\bar{e}} + \eta_\theta \theta_{\bar{e}})\frac{\partial e_*}{\partial c_u} + \eta_\theta \theta_w \frac{\partial w_*}{\partial c_u}\right][v(c_e) - u(c_u)] \\
&\quad + \mu\left\{\eta \frac{\partial w_*}{\partial c_u} - (1-\eta) + \left[(\eta_e + \eta_{\bar{e}} + \eta_\theta \theta_{\bar{e}})\frac{\partial e_*}{\partial c_u} + \eta_\theta \theta_w \frac{\partial w_*}{\partial c_u}\right](w_* - c_e + c_u)\right\}. \qquad (2.83)
\end{aligned}
$$

Dividing (2.82, 2.83) respectively by $\eta\mu$ and $(1-\eta)\mu$ and subtracting the resulting equations, we find the Baily-Chetty variants

$$
\begin{aligned}
&\frac{u'(c_u) - v'(c_e)}{\mu} \\
&= (\eta_e + \eta_{\bar{e}} + \eta_\theta \theta_{\bar{e}})(w_* - c_e + c_u)\left(\frac{1}{\eta}\frac{\partial e_*}{\partial c_e} - \frac{1}{1-\eta}\frac{\partial e_*}{\partial c_u}\right) \\
&\quad + \left[\eta_\theta \theta_w \frac{v(c_e) - u(c_u)}{\mu} + \eta + \eta_\theta \theta_w (w_* - c_e + c_u)\right]\left(\frac{1}{\eta}\frac{\partial w_*}{\partial c_e} - \frac{1}{1-\eta}\frac{\partial w_*}{\partial c_u}\right) \\
&\quad + (\eta_{\bar{e}} + \eta_\theta \theta_{\bar{e}})\frac{v(c_e) - u(c_u)}{\mu}\left(\frac{1}{\eta}\frac{\partial e_*}{\partial c_e} - \frac{1}{1-\eta}\frac{\partial e_*}{\partial c_u}\right), \qquad (2.84) \\
&= \eta_e (w_* - c_e + c_u)\left(\frac{1}{\eta}\frac{\partial e_*}{\partial c_e} - \frac{1}{1-\eta}\frac{\partial e_*}{\partial c_u}\right) \\
&\quad + \left[\eta_\theta \theta_w \frac{v(c_e) - u(c_u)}{\mu} + \eta + \eta_\theta \theta_w (w_* - c_e + c_u)\right]\left(\frac{1}{\eta}\frac{\partial w_*}{\partial c_e} - \frac{1}{1-\eta}\frac{\partial w_*}{\partial c_u}\right) \\
&\quad + (\eta_{\bar{e}} + \eta_\theta \theta_{\bar{e}})\left[\frac{v(c_e) - u(c_u)}{\mu} + (w_* - c_e + c_u)\right]\left(\frac{1}{\eta}\frac{\partial e_*}{\partial c_e} - \frac{1}{1-\eta}\frac{\partial e_*}{\partial c_u}\right). \qquad (2.85)
\end{aligned}
$$

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