Implications of Uncertainty for Optimal Policies

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Overview

Mutual implications between:

- Optimal dynamic fiscal policy

  (constrained-efficient, information and/or commitment friction)

- Broader view of uncertainty

  (risk + ambiguity/Knightian uncertainty, aversion to both)

Focal setting: dynamic Mirrlees with uncertainty about DGP

Motivation

Standard approaches:

- risky future skills + agents & government certain about DGP

Drawbacks:

1. Difficult to find empirical support for DGP certainty
    - substantial uncertainty about macro and micro variables (Bloom 2014)
    - pre-tax income distributions change significantly, often (Piketty, Saez, Zucman 2018)
2. Conclusions sensitive to specifics of DGP, etc.
    - $\frac{1-F(\theta)}{\theta f(\theta)}$ drives top marginal labor tax rates 20↔65% (Golosov, Troshkin, Tsyvinski 2016)
3. Welfare costs of ignoring broader uncertainty

Motivation

Standard optimal policies:

- once-and-forever
- history-dependent, complex
- complete

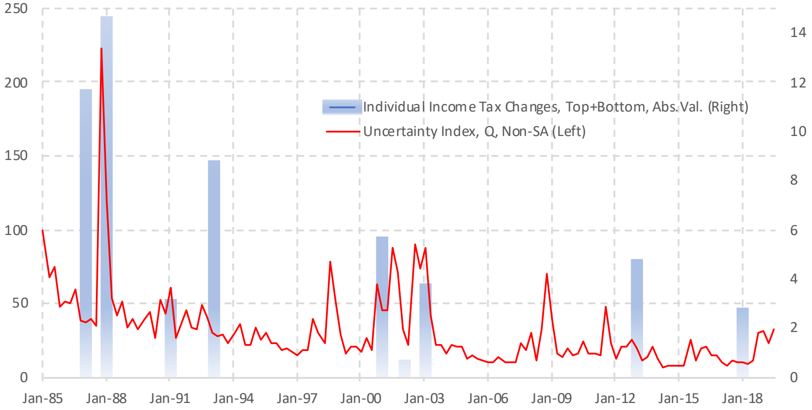
Commonly-observed policies:

- periodic reforms (especially income taxes)
- often ignore history, simplified
- incomplete (at least somewhat)

Without certainty, can these be optimal?

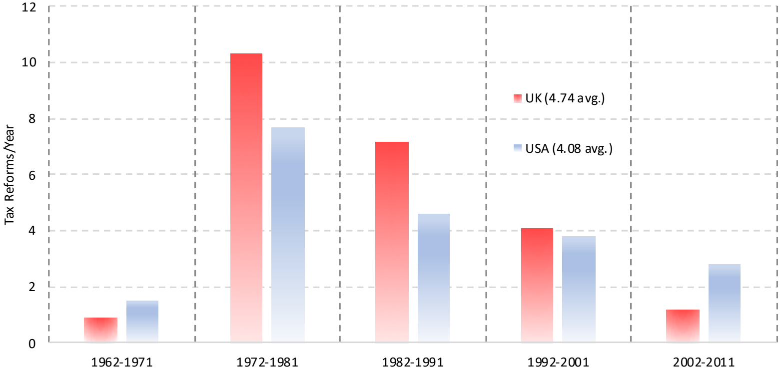
Can be achieved by competitive insurance markets?

Reforms vs. uncertainty:



Sources: IRS SOI and Baker, Bloom, Davis (2012)

Periodic reforms:



Source: IMF Tax Policy Reform Database and Amaglobeli et al. (2018)

Preview of results

Periodic reforms optimal

- uncertainty ≈ “endogenous no-commitment”
- even with full commitment, information symmetry
  (extends to private skills, beliefs)

Loss of history dependence

- gov’t promise-keeping constraint slack after reform

Incomplete / simplified policies,

- $T < \infty$: no full backward induction for promise utility
- but: linear policies generically sub-optimal

+ Normative approach with meaningful role for social insurance

- uncertainty + private info ⇒ CE not efficient

Related literature

- Optimal dynamic social insurance & redistribution
  - Standard dynamic settings: Kocherlakota (2010), Farhi Werning (2013), Golosov Troshkin Tsyvinski (2016)
  - Added frictions from labor mkt, human capital: Scheuer Werning (2016), Stantcheva (2017), Makris Pavan (2018)
  - Inability to commit by gov't: Farhi Sleet Werning Yeltekin (2012)
- Decentralization + crowding out private insurance
  - Golosov Tsyvinski (2007), Acemoglu Simsek (2012)
- Relaxing assumption of certainty of DGP
  - Kocherlakota Phelan (2009): uncertain gov't, endowment shocks, public policies can't improve on CE
  - Bhandari (2015): risk sharing in Hansen Sargent (2001) setting
- Formalism: Epstein Schneider (2003), Hansen Sargent (2001), Bergemann Morris (2013)

Outline

- Model
- Periodic reforms
- Generalizations
  - more general beliefs, preferences
  - private skills, beliefs
  - lack of commitment by agents
- Inefficiency of CE

Model

Illustrative example

No intrinsic value, but easy to understand

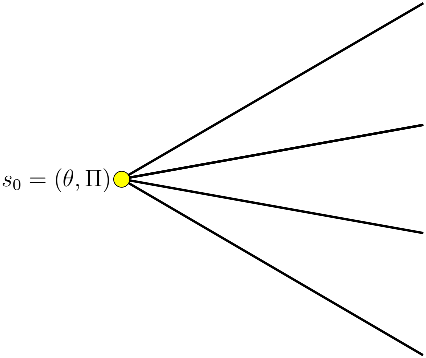
Paper's contribution: this extends to general optimal policies

- Time: $t = 0, 1, 2$
- Agents: $i = A, B$
- Idiosyncratic shocks $s_{i,t}$: unknowable finite stochastic process
- $s_{i,t} \equiv (\theta_{i,t}, \Pi_{i,t+1})$:
  - skill $\theta_{i,t} \to$ effective labor $z_{i,t} = \theta_{i,t} l_{i,t}$
  - belief $\Pi_{i,t+1} \equiv$ set of distributions over s^{t+1}

    (agnostic about updating/learning: Π in s for convenience)

- Allocation: $C \equiv \left\{c_t\left(s^t\right), z_t\left(s^t\right), k_{t+1}\left(s^t\right)\right\}_{t=0,1,2}$

Illustrative example



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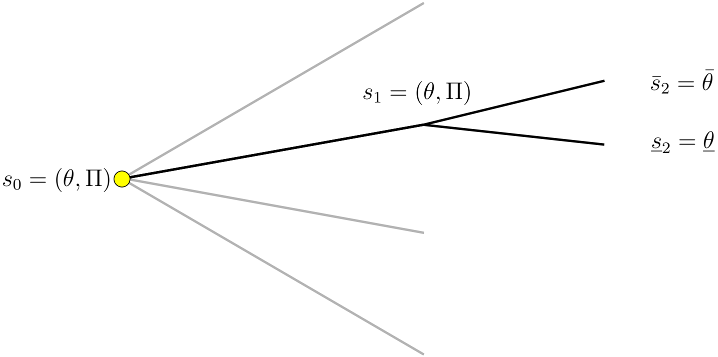

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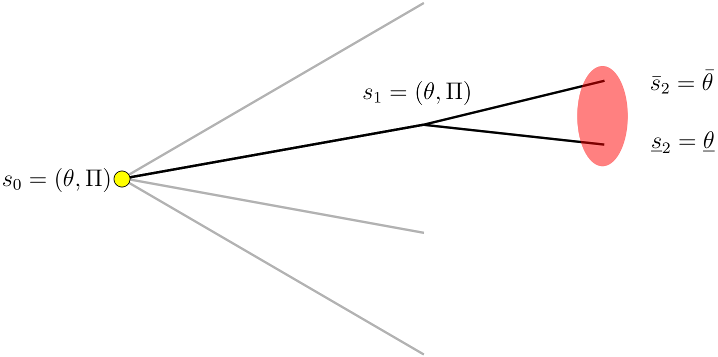

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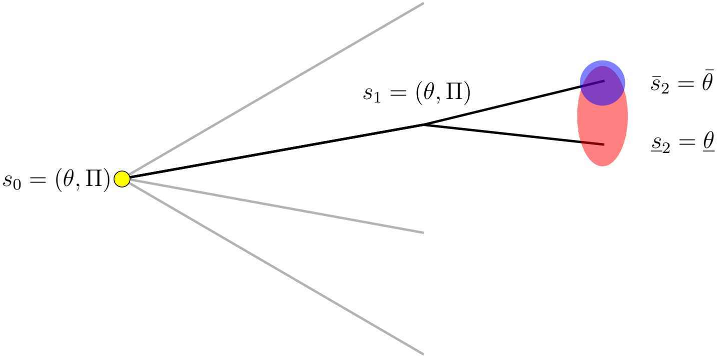


Aversion to risk, uncertainty

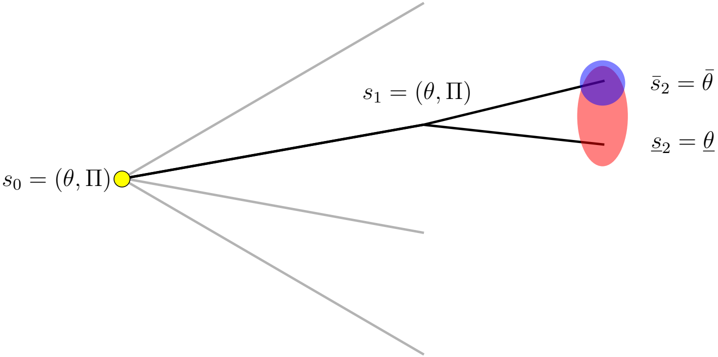
Assume recursive utility (does not have to be maxmin):

$$U_{i,t}\left(C|\, s^t\right) \equiv u\left(c_{i,t}\left(s^t\right), \frac{z_{i,t}\left(s^t\right)}{\theta_{i,t}}\right) + \beta \inf_{\Pi_{i,t+1}} \mathbb{E}_{\pi_{i,t+1}}\left[U_{i,t+1}\left(C|\, s^{t+1}\right)\Big|\, s^t\right]$$

- $\pi_{i,t+1} \in \Pi_{i,t+1}$
- axiomatization, recursive repr'n: Epstein-Schneider(2003)

Results more general, e.g.: ... $sup_{\Pi_{i,t+1}} \mathbb{E}_{\pi_{i,t+1}}[...]$

Assumption on beliefs:



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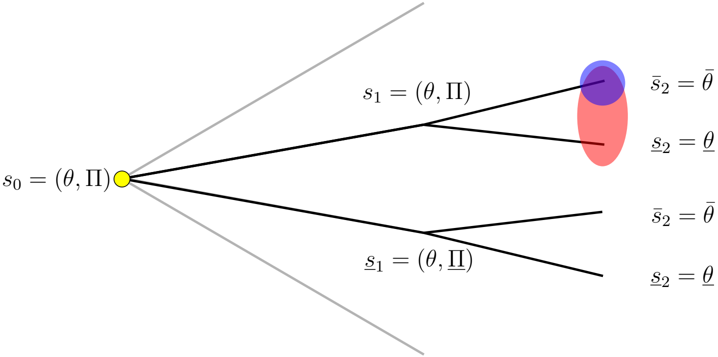

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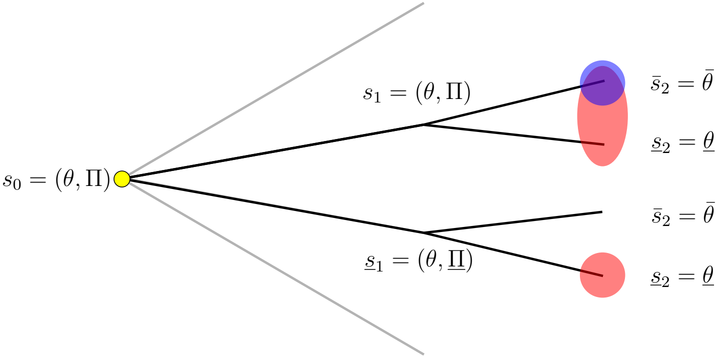

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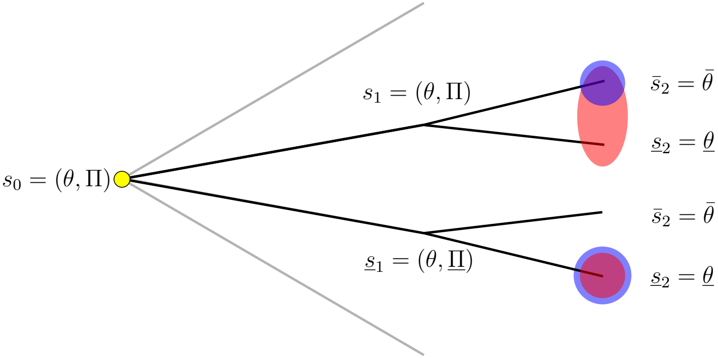

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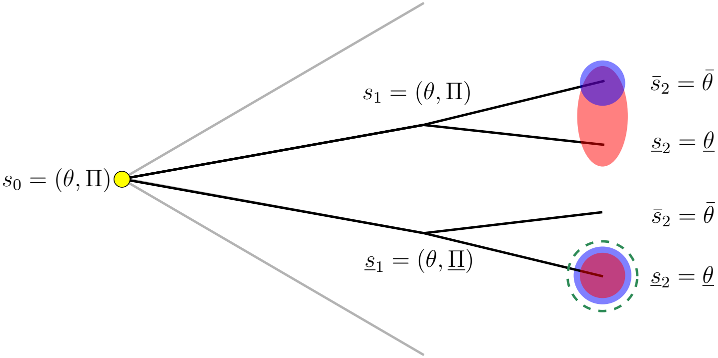


Assumption on beliefs: agree on feasible path

- Obvious example - economy's "worst" path
- For any belief $\pi_{i,t+1}$, there is $\underline{\pi}_{i,t+1}$:
  - same marginal distribution of θ
  - but marginal of Π has unit weight on $\underline{\Pi}$
- Notice:
  - DGP not required to place weight on this path
  - any (heterogeneous) marginals of θ allowed
  - can be relaxed significantly

Periodic reforms

Government

(symmetric information, full commitment)

- C^* is efficient given Pareto weights η_i if

$$C^*(s_0) \in \arg\max_C \sum_i U_{i,0}(C|\, s_0)\, \eta_i$$

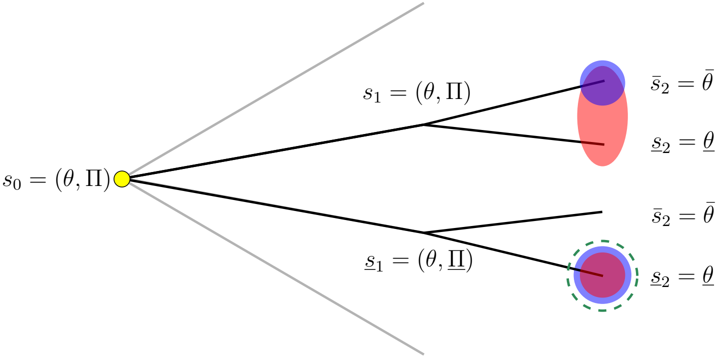
s.t. non-negativity and feasibility:

$$\sum_i c_{i,t}\left(s^t\right) + K_{t+1}\left(s^t\right) \leq f\left(K_t\left(s^{t-1}\right), Z_t\left(s^t\right)\right), \;\; \forall t, s^t \geq s^0$$

- ex-post feasibility reflects (heterogeneous) uncertainty
- government knows no more than agents

if certainty: C^* once-and-forever, history dependent, complex

Periodic reform mechanism:



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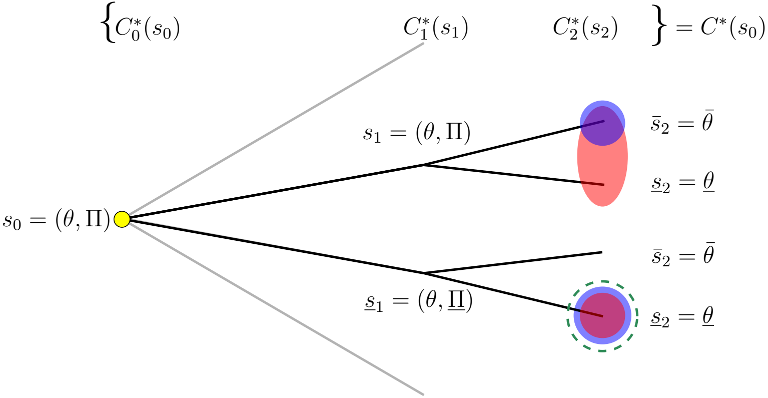

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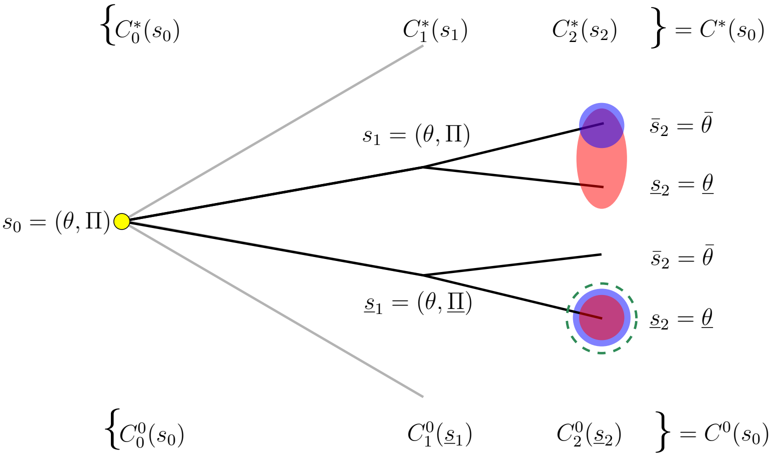

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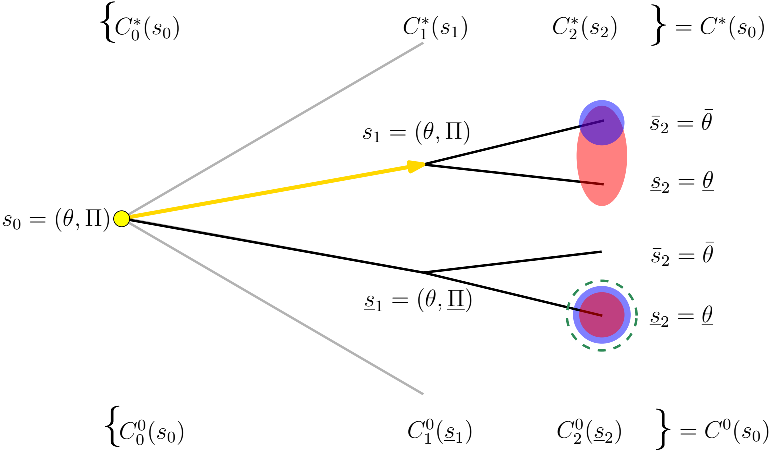

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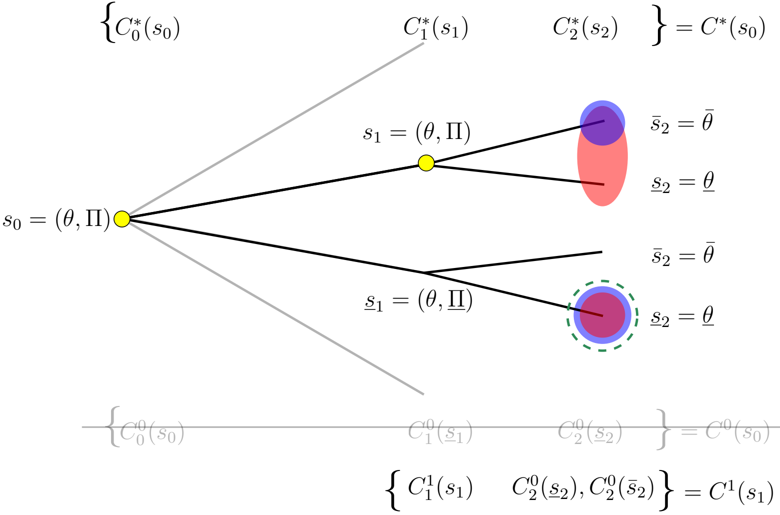

Proposition (periodic reforms):

Given efficient C^*, there is sequence $\{C^t\}_{t=0}^T$, where $C^t = \left\{c_\tau^t, z_\tau^t, k_{\tau+1}^t\right\}_{\tau=t}^{t+1}$ are incomplete and

$$U_{i,0}\left(C^0 \middle| s^0\right) = U_{i,0}\left(C^* \middle| s^0\right) \quad \forall i,$$

$$U_{i,0}\left(C_0^0, \left(C_t^1\right)_{t=1}^T \middle| s^0\right) \geq U_{i,0}\left(C^0 \middle| s^0\right) \quad \forall i,$$

$$U_{i,1}\left(C_1^1, \left(C_t^2\right)_{t=2}^T \middle| s^1\right) \geq U_{i,1}\left(C^1 \middle| s^1\right) \quad \forall i,$$

$$\ldots$$

Mechanism:

- uncertainty aversion & sufficient belief overlap ⇒ need only t & worst-case $t+1$
- when worst not realized ⇒ reform $t+1$ & worst-case $t+2$...
- generalization of incomplete contract ideas (e.g. Mukerji 1998, Zhu 2016)

Proof by constructing incomplete C^t

- Start $C_0^0 = C_0^*$, set C_1^0 to worst-case C_1^*
  - i.e. with $\underline{\Pi}_2$
    (C^0 not fully state contingent, depends only on s^0 and θ_1)

- At $t = 0$, all agents : $C^* \sim C^0$
  - $\inf_{\Pi_{i,1}} \mathbb{E}_{\pi_{i,1}} \left[U_{i,1} (.) \middle| s^0 \right]$ attains if $\pi_{i,1}$ puts all weight on $\underline{\Pi}_2$
  - sufficient belief overlap $\Rightarrow$ such $\pi_{i,1}$ exist in $\Pi_{i,1}$

- At $t = 1$, if $\underline{\Pi}_2$ not realized: C_1^0 can be improved to C_1^1 & worst-case C_2^1, and so on..
  - C_1^0 still feasible
  - ..so acts like *endogenous outside option (fallback)*

Discussion

Simple algorithm for constructing optimal allocations

- if not $\underline{\Pi}$: reform will welfare-improve
- fallback: previous allocation C^{t-1}
  (form of endogenous lack of commitment)
- dependence on history only via promise-keeping

Incomplete, simpler optimal policies

- lose history whenever reform Pareto-improves
  (e.g. whenever C^t can be constructed by backward induction)
- limited dependence on future shocks, distributions

Optimal Reform Problem

Given previous policy C^{t-1}, reform to

$$C^t\left(s^t, C^{t-1}\right) \in \arg\max_{C^t} \sum_i U_{i,t}\left(C^t \middle| s^t\right)\eta_i$$

s.t. non-negativity, feasibility $\tau = t, t+1$, $s^\tau \geq s^t$

$$\sum_i c_{i,\tau}^t\left(s^\tau\right) + K_{\tau+1}^t\left(s^\tau\right) \leq f\left(K_\tau^{\tau-1}\left(s^{\tau-1}\right), Z_\tau^t\left(s^\tau\right)\right),$$

and promise-keeping $\forall i$

$$U_{i,t-1}\left(C_{t-1}^{t-1}, \left(C_\tau^t\right)_{\tau=t}^T \middle| s^t\right) \geq U_{i,t-1}\left(C^{t-1} \middle| s^t\right)$$

→ algorithm for simplified characterization/computation

Optimal Reform Problem

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$$C^t\left(s^t, C^{t-1}\right) \in \arg\max_{C^t} \sum_i U_{i,t}\left(C^t \middle| s^t\right)\eta_i$$

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and promise-keeping $\forall i$

$$U_{i,t-1}\left(C^{t-1}_{t-1}, \left(C^t_\tau\right)^T_{\tau=t} \middle| s^t\right) \geq U_{i,t-1}\left(C^{t-1} \middle| s^t\right)$$

→ algorithm for simplified characterization/computation

Dynamic consistency?

Preferences *dynamically consistent* in natural sense :

- If $C, \tilde{C}$ coincide at t and for all $s^{t+1} \geq s^t$

$$U_{i,t}\left(C|\, s^{t+1}\right) \leq U_{i,t}\left(\tilde{C}\middle|\, s^{t+1}\right),$$

  then

$$U_{i,t}\left(C|\, s^{t}\right) \leq U_{i,t}\left(\tilde{C}\middle|\, s^{t}\right)$$

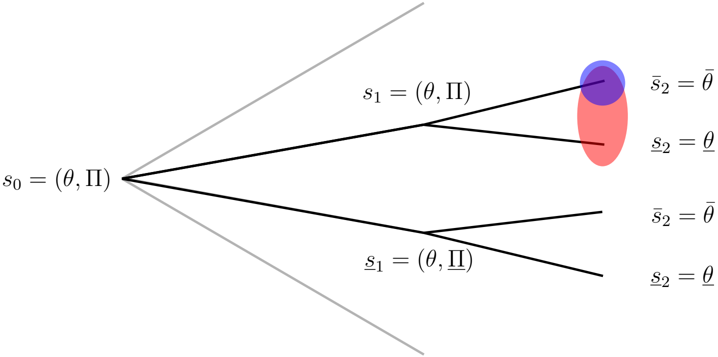
  - immediate from recursive rep. of $U_{i,t}$
  - current beliefs are not allocation dependent

- Same notion as:
  - Epstein-Schneider(2003), Maccheroni, Marinacci, Rustichini(2006), Klibanoff, Marinacci, Mukerji(2005), etc.

- Implies:
  - agents can find ex-ante solution by backward induction (weaker/more policy-relevant, e.g. Hansen-Sargent 2001 multiplier)

Generalizations

More general beliefs, preferences:



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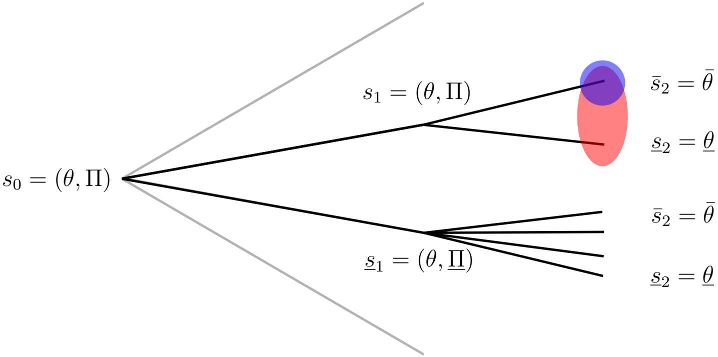

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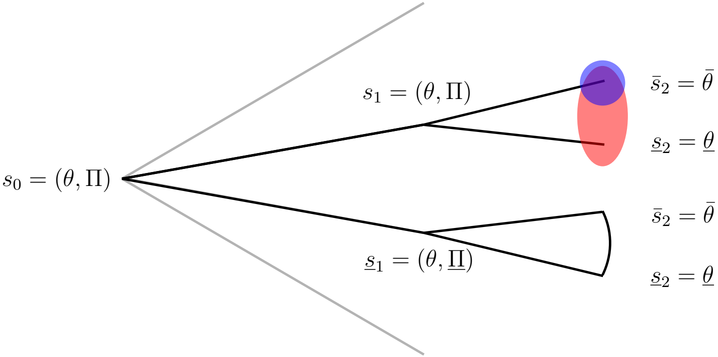

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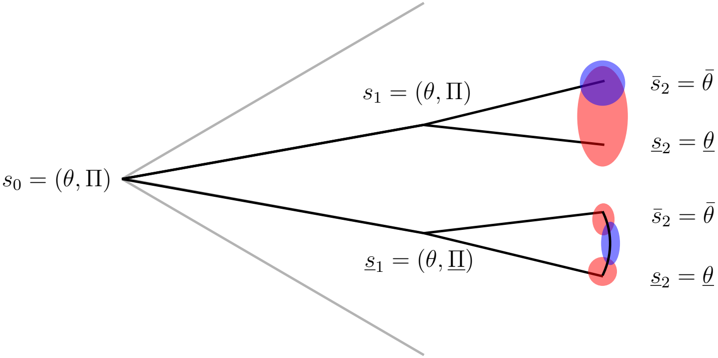

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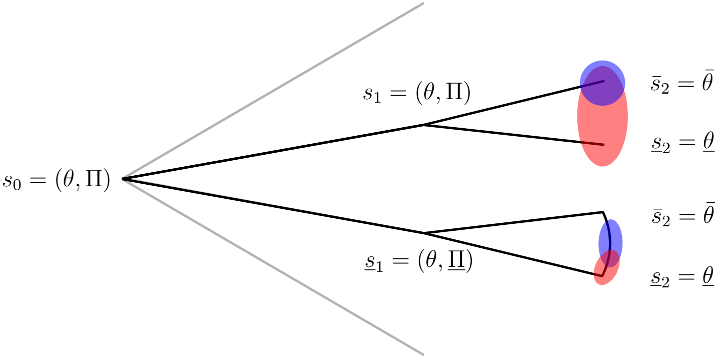

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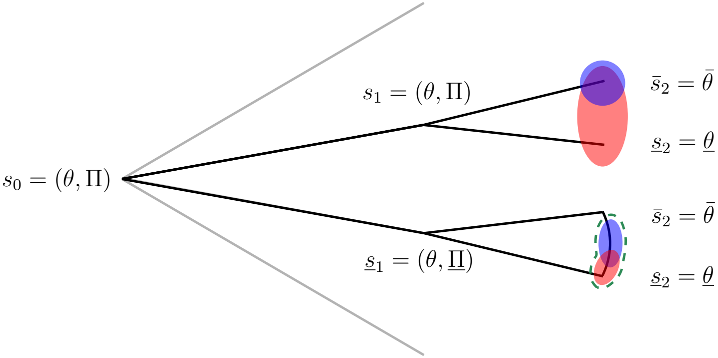


Private skills, beliefs:

- Assumption on beliefs: agree on feasible path (as before)
- Obvious example - economy's "worst" path (as before)
- New: well-defined "worst" $\leftarrow$ coincidence between:
  - worst in resources
  - worst in subjective-continuation-utility
- Additional assumption: weak monotonicity of allocations
  - weakly worse off if all others certain to report $\underline{\theta}$ at t, $t+1$, ..

Agents with outside options:

- Each agent has outside option $\underline{U}_{i,t}\left(s_i^t\right)$
    - exogenous lack of commitment from agents
- Recall Reform Problem:
    - promise-keeping is a form of endogenous self-enforcement
    - then straightforward to handle exogenous self-enforcement:

$$U_{i,t}\left(C^t \middle| \hat{s}^{t-1}, s_i^t\right)(\sigma^*) \geq \underline{U}_{i,t}\left(s_i^t\right)$$

Inefficiency of CE (with private shocks)

Source of inefficiency

- Competitive firms, contract one-to-one with agents:
  - buy k_0, employ $z_{i,t}$, produce $f(k_{i,t}, z_{i,t})$, return $c_{i,t}$
  - adopt agents' beliefs $\Pi_{i,t+1}$
  - reinterpretation: agents have direct access to f, securities markets

- Even if all Arrow-Debreu securities available:
  - securities contingent on idiosyncratic reports not traded in CE
  - immediate from arbitrage vs. risk-free bonds

    (e.g. Acemoglu Simsek 2012, Golosov Tsyvinski 2007)

- $\Rightarrow$ CE not efficient in general

Only risk-free bonds in equilibrium

Lemma. Securities contingent on idiosyncratic reports $\hat{s}_i^t$ are not traded in CE.

- Security $a\left(\hat{s}_i^t\right)$ pays if agent i reports $\hat{s}_i^t$
- Suppose $a\left(\hat{s}_i^t\right)$ costs strictly less than risk-free bond:
  - i buys ∞ $a\left(\hat{s}_i^t\right)$ and sells ∞ risk-free bonds, reports $\hat{s}_i^t$ at t
  - i nets ∞ profit, sellers of $a\left(\hat{s}_i^t\right)$ guaranteed to lose $\rightarrow\leftarrow$

$\Rightarrow$ only risk-free bonds traded in CE

CE inefficiency: simple example

$\Pi_{A,1} = \left\{\underline{\pi}_{A,1}, \bar{\pi}_{A,1}\right\}$:

- $\underline{\pi}_{A,1}$: both certain to draw $\underline{\theta}$ at $t \geq 1$
- $\bar{\pi}_{A,1}$: both certain to draw $\bar{\theta}$ at $t \geq 1$
- believes $\Pi_{B,1} = \left\{\underline{\pi}_{B,1}, \bar{\pi}_{B,1}\right\}$

$\Pi_{B,1} = \{\bar{\pi}_{B,1}\}$:

- $\bar{\pi}_{B,1}$: both certain to draw $\bar{\theta}$ at $t \geq 1$
- believes $\Pi_{A,1} = \{\bar{\pi}_{A,1}\}$

CE inefficiency: simple example

Planner:

- transfer consumption to A if both draw $\underline{\theta}$ at $t = 1$
- IC satisfied: B does not believe A will draw $\theta_{A,1} = \underline{\theta}$

CE:

- A would have to insure by purchasing risk-free bond
- $\Rightarrow U_{A,0}$ below efficient

Planner insures risk and ambiguity

But notice: in CE, nothing prevents decentralized periodic reforms, history independence, incompleteness

Takeaways

Takeaways

Optimal fiscal policies, without certainty about DGP:

- Simplified, more realistic optimal policies
  - reformed periodically, incomplete, not fully history dependent
- Simplified algorithm to compute optima
  - computes policy period-at-a-time
  - no full-backward-induction for promises
- Normative approach with meaningful role for social insurance
  - beyond crowding out private insurance

Thank you

Ancillary slides

Risk and heterogeneity

Source: stochastic output $f_t : (k_t, z_t) \rightsquigarrow y_t$

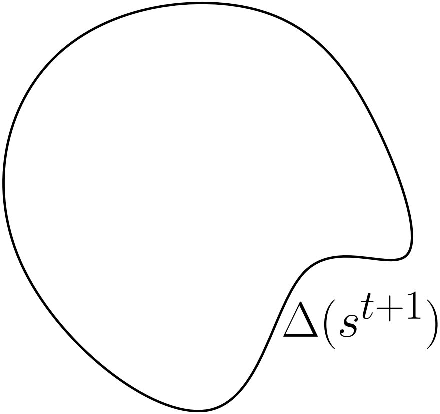
- Examples (depending on application):
  - random variable $f_t(\cdot, \cdot)$
  - deterministic $f(\cdot, \cdot)$, $z_t = \theta_t l_t$ with stochastic θ_t
  - deterministic $f = R_t k_t + \theta_t l_t$, stochastic θ_t,R_t
- Second or third options: idiosyncratic $s_t = (\theta_t, \Pi_{t+1})$
  - skill $\theta_t \in \Theta \equiv \left\{\underline{\theta} < \ldots < \bar{\theta}\right\}$
  - beliefs Π_{t+1} about s^{t+1}

    (agnostic updating/learning: for convenience Π in s)

Uncertainty in macro: Hansen-Sargent example

- agent i at t has statistical model $\pi^*_{i,t+1}$ of θ_{t+1}
  (focus on θ_{t+1}, "too difficult" to eliminate any π_{t+2})
- distrusts it, considers "nearby" models π_{t+1}:

$$\Pi^{HS}_{i,t+1} \equiv \left\{ \pi_{t+1} | \, d\left(\pi^*_{i,t+1}, \pi_{t+1}\right) \leq \epsilon \right\}$$



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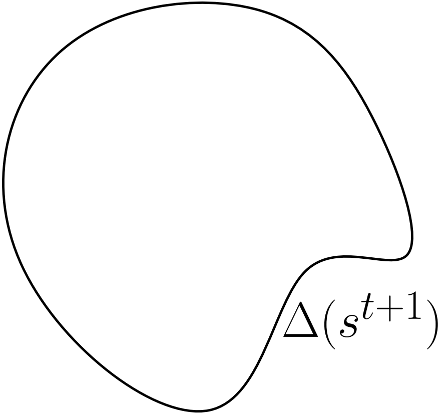

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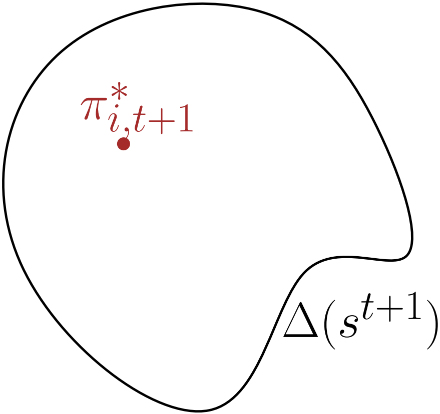

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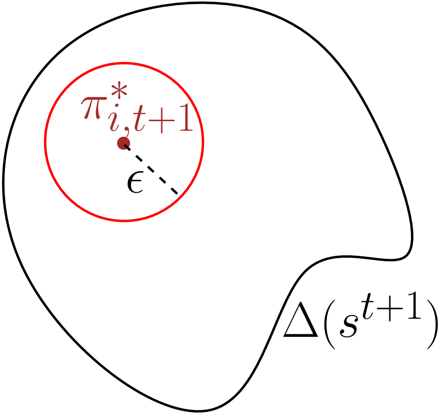

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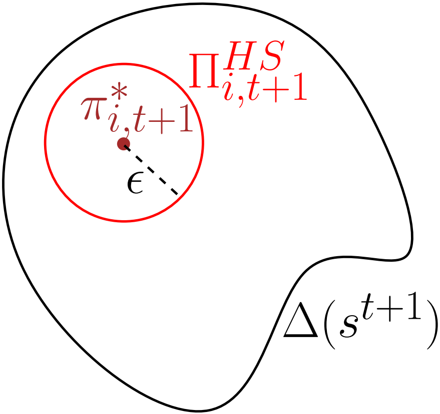

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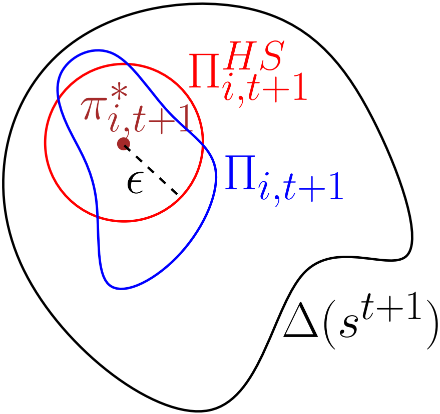
- distrusts it, considers "nearby" models π_{t+1}:

$$\Pi^{HS}_{i,t+1} \equiv \left\{ \pi_{t+1} | \, d\left(\pi^*_{i,t+1}, \pi_{t+1}\right) \leq \epsilon \right\}$$



Uncertainty in macro: Hansen-Sargent example

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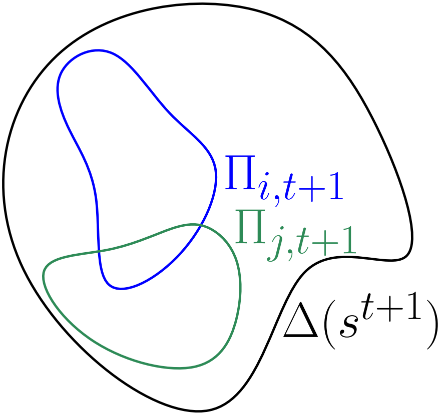

Periodic reforms in equilibrium

- At $t = 0$, agent i solves for fully continent allocation

$$C_i = \left\{c_{i,t}\left(s^t\right), z_{i,t}\left(s^t\right), k_{i,t+1}\left(s^t\right), b_{i,t+1}\left(s^{t-1}\right)\right\}_{t=0}^{T}$$

  - given risk-free bond prices $\{Q\left(s^t\right)\}_{t=0}^{T-1}$

Proposition: For any $C = \{C_i\}_{i=1}^{N}$, there exist incomplete allocations $\{C^t\}_{t=0}^{T}$ such that

$$U_{i,0}\left(C|\, s_0\right) = U_{i,0}\left(C^0\middle|\, s_0\right)\ \forall i, s_0$$

- Periodic reforms decentralized: each C^t designed assuming that all agents receive worst beliefs $\underline{\Pi}_{t+2}$ and worst shock $\underline{\theta}$ at $\tau \geq t+2$